STATIC ANALYSIS OF
FRAMED STRUCTURES

By

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PREFACE

The material presented in this report is the outgrowth of three years of work collecting computer programs which are of benefit to the structural engineer. This work was begun in January, 1965, when I first experienced the advantage of using the electronic computer in structural engineering, while enrolled at Oklahoma State University in Civil Engineering 4B4 under Dr. Winfred O. Carter. I have reviewed hundreds of structural programs in the intervening period, and I have selected some of the best in order to demonstrate what can be done with them. The field of structural programming has opened in the last ten years, and it is rapidly being accepted by professional engineers.

I wish to express my indebtedness and gratitude to the following persons:

To Mr. Jack Frederickson and Mr. David Benham, both of Benham-Blair and Affiliates - Consulting Engineers, Oklahoma City, Oklahoma, for their continued encouragement and help in pursuing my interests in computer work in civil engineering.

To Dr. David MacAlpine for his guidance in my civil engineering education, and his personal confidence in my ability to achieve my goals.

To my father, Mr. Beverly C. D. Edwards, for an unbelievable amount of inspiration.

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CHAPTER I

INTRODUCTION

Like all of the applied sciences, the field of civil engineering is in a constant state of change and improvement. As new methods are developed by the scientist, they are passed on to the engineer for practical application. New products are developed for the architect to include his design, and incorporate into the specifications. Methods have been revised and improved in most phases of all branches of the engineering field.

Unlike the rest, structural engineering has not fully enjoyed the prosperity of new development of ideas. The classical methods of structural analysis have remained in use, almost up to the present, for want of any faster, cheaper, safer, or more accurate means of solving for the stresses and strains in highly indeterminate structures. One of the greatest advancements came with the advent of the Hardy Cross method of distributing the moments in continuous beams and rigidly framed structure This brought a great savings in time spent on frame analysis. Moment distribution was easy to apply, and this efficient approach came into common acceptance throughout the field of analysis. Other modifications and short cut methods have been discovered, and put into use in recent years. Plastic design of continuous beams and rigid frames is growing in acceptance in design offices, although it still has its stubborn opponents. Many short cut methods such as the cantilever and portal methods of wind load analysis still leave something to be desired, however.

The cantilever method is not as accurate as the slope deflection method in evaluating the moments and shears in a multi-story rigid fram under wind load, but it is often used because the solution is so much easier to obtain. The quality of the answers obtained with the approximate methods are sometimes questioned. If the answers obtained through the cantilever method are higher than those of the slope deflection meth this may mean a waste of building materials in the final design. Likewise, low answers using the approximate solution may lead to a design wi a low factor of safety. It goes without saying that the most accurate answer is preferred. Exact analysis of highly indeterminate structures is usually more time consuming than approximate analysis, and time is money to the engineer and his client. Because of lack of time or abilit designers continually have been forced to make many simplifying assumptions in their analysis work. No one can say how many structures have been excessively over or under designed because of these simplifying assumptions. Add to this the fact that many outright mistakes in the analysis are never detected, and one sees that structural engineering ha: its problems, like everything else.

The entrance of the electronic computer into the field of structur: engineering will not act as a panacea to cure all of these ills, but it will help considerably.

> "The first successful digital computer was built by a French mathematician, Blaise Pascal, in 1642. It was, of course, mechanical in nature; it employed wheels on which were placed the decimal digits 0 through 9 and which were rotated on an axle by means of gears and ratchets. It was the

Forerunner, through much development, of the adding machine."[1]

The British began work on the antecedent of the modern stored-program computer in 1812, with the work of Charles Babbage. With his "differenc machine", Babbage was successful in obtaining 6-digit accuracy in the roots of polynomials, but his work was discontinued, for various reasons. In 1944, interest was revitalized at Harvard University, when Dr. Howard Aiken designed the first large-scale general-purpose digital computer, which was built by the International Business Machines Corporation (IBM). From then until the present, the progress and improvement of the electronic computer has been dramatic. The various stages in the development of the computer in the last two decades are incidental to this discussion but since about 1957 structural engineers have begun to take a serious look at its potential in structural problem solving.

Numerical methods of solving structural problems have been defined in this last decade, and further development and utilization is now in an expotentially increasing growth pattern. The early feelers went out through scholars in the late 1950's, but since about 1961 commercial interest has seized upon the idea of rapid structural analysis, principally through the introduction of the IBM 1620 computer on the market. Skepticism still ranged wide in the field of computer analysis of indeterminate structures, at first. Many were, and still are, reluctant to retire from the manual methods of analysis. The same classical methods of slope deflection, moment distribution, virtual work, flexibility and the stiffness method are used to solve structural problems on the computer, but

[1] Southworth and Deleeuw, Digital Computation and Numerical Methods, (McGraw-Hill, 1965), p.5.

their solution time is greatly accelerated. Once a numerical method or program is precisely formulated for a structural problem, the speed and accuracy of the computer will far outdistance the manual approach to solution. This certainly is not without its exceptions, but for most conventional problems in analysis, the computer may be readily applied with success. This does not close the door to the area of design, but this has not been as fully developed as computer programming of structural analysis problems. Design programs are beginning to appear, as will be shown later, but since design is often more of a creative art than a science, its procedures are sometimes harder to define and translate into a logical flow pattern, necessary for a computer program.

This report is a study of the work that has been done in the past ten years (1957-1967) in the area of electronic computation in structural engineering. It would almost be impossible to discuss every program that has been written in this period, since most are intentionally withheld from publication, because of their commercial value. Programming has been done in several general areas of framed structures, and typical programs are presented from each. This report deals with framed structures such as continuous beams, rigid frames, and trusses, in two and three dimensions. No attempt is made to investigate plates and shells. The programs presented were compiled over a period of three years, but they date back to as early as 1960. In addition to programs of a general nature, such as a space frame program for the analysis of any three dimensional rigid frame, some specialized programs have been included. An example of such a special purpose program would be the program presented for analysis of gabled frames with linearly or parabolically haunched members. The general purpose programs are the most widely applicable, and usually the most involved and lengthy to write, so they are the most significant to the study. Special purpose programs such as those presented in Chapter VI, do serve their purpose for special problems which are easily programmed. Those special purpose programs presented are only a small portion of the many possibilities which may arise for programming a particular chain of logical decisions or calculations involved in a problem which is to be repeated many times.

An outline of each program is presented, along with a discussion of its merits (or lack of such), and a sample problem is worked with each program. The actual print-out of the Fortran source program may be referred to under separate cover. All programs are written in either Fortran II for the IBM 1620 computer, or Fortran IV for the I.B.M. 7040 computer. The languages are almost identical, and the greatest differences lie in the input/output statements. These programs could easily be converted to use on the computers manufactured by General Electric. It is assumed that the reader has a working knowledge of the IBM Fortran language.

CHAPTER II

COMPUTER ANALYSIS OF FRAMES BY MOMENT DISTRIBUTION

Analysis of Single Story Orthogonal Rigid Frames

The method of moment distribution, or the "Cross-Method", was first developed by Hardy Cross in the period of 1922 to 1924, and presented to his students at the University of Illinois from 1929 to 1932. Since its publication by Cross, it has come into wide acceptance It probably is the most commonly used method of frame analysis.

> "Undoubtedly one of the reasons for its great appeal to engineers is the fact that each step in the calculations can be interpreted readily in terms of the physical behavior of the structure being analyzed."[1]

The many merits of moment distribution have made it very popular as a manual method of analyzing indeterminate structures, but let us now concern ourselves with applying it with the aid of the computer.

> "The method is suited to digital computer programming, because of the repetitive or cyclical nature of the calculations and the fact that the calculations are well-behaved. This means that the programmer normally does not have to become concerned with questions pertaining to loss of numerical accuracy."[1]

The calculations can be terminated when any desired degree of accuracy has been obtained through the distribution.

The first step in programming of moment distribution is to input the data pertaining to the geometry of the structure. This first progr

[1] James M. Gere, Moment Distribution, (D. Van Nostrand Co., Inc.,) p.vii

is designed to distribute the moments in a continuous beam or in a building frame, one story at a time. For one story of a single or mul story orthogonal rigid frame, this includes reading into the computer memory the values of the number of joints, the accuracy tolerance for the distribution, the properties of the beams (length, moment of inert uniform load applied to beam), the number of columns, and the properti of the columns. A single story, having beams and columns tying in from below and/or above, may be made into a continuous beam (no columns) by giving the imaginary columns a moment of inertia of zero. Thus the pro gram can be used for continuous beams or a single story of a multi-bay rigid frome by juggling the input. This will be made clear in the sample problem. Next, the fixed end moments, stiffness factors, carry-over factors, and distribution factors for the various joints are calculated.

> "The next step in the main program is the distribution of moments. This requires the programming of one complete cycle of calculations (that is, one cycle of moment distribution) which is to-be executed repeatedly until a specific degree of accuracy is obtained. For example, the programmer can specify that the cycle is to be repeated until the ratio of the last carry-over moment at a particular joint to the currently calculated value of the moment at that same joint is less than a specified (small) quantity."[2]

This is illustrated in the flow chart of the program.

In this particular program, the end moments are now punched out. An easy addition to the program would be computation of end shears on the members.

If nonprismatic members are present, the altered stiffness and carry-over factors may be inserted.

[2] James M. Gere, Moment Distribution, (D.Van Nostrand Co., Inc.), p.24

This program is handy for use on a small computer for finding the moments in a multi-story building due to live load, taking the building a story at a time. It also is convenient for use with continuous beams under varying uniform loads, or single story frames without side sway.[3]

It has a few disadvantages, however:

1) Side sway is not considered in the case of a frame.

2) The only loading condition is for uniform load placed on the beams.

3) A continuous beam is almost as easy to do by long-hand.

4) It only takes a multi-story frame a story at a time.

Its principal advantage is that it requires only a small memory computer such as the IBM 1620, when a larger machine is not available for use of a more comprehensive program. The input data is relatively easy, but the joint and member numbering order should be noted in the sample.

A flow chart of the program appears next, and then a sample problem is worked.

[3] This program was originally written for Civil Engineering 4B4, at Oklahoma State University, in April, 1965, under the direction of Dr. W. O. Carter. It was written by the author, in collaboration with Mr. Donald Kerr and Mr. P. L. Chen.

FLOW CHART FOR MOMENT DISTRIBUTION OF SINGLE-STORY OF ORTHOGONAL RIGID FRAME OR CONTINUOUS BEAM

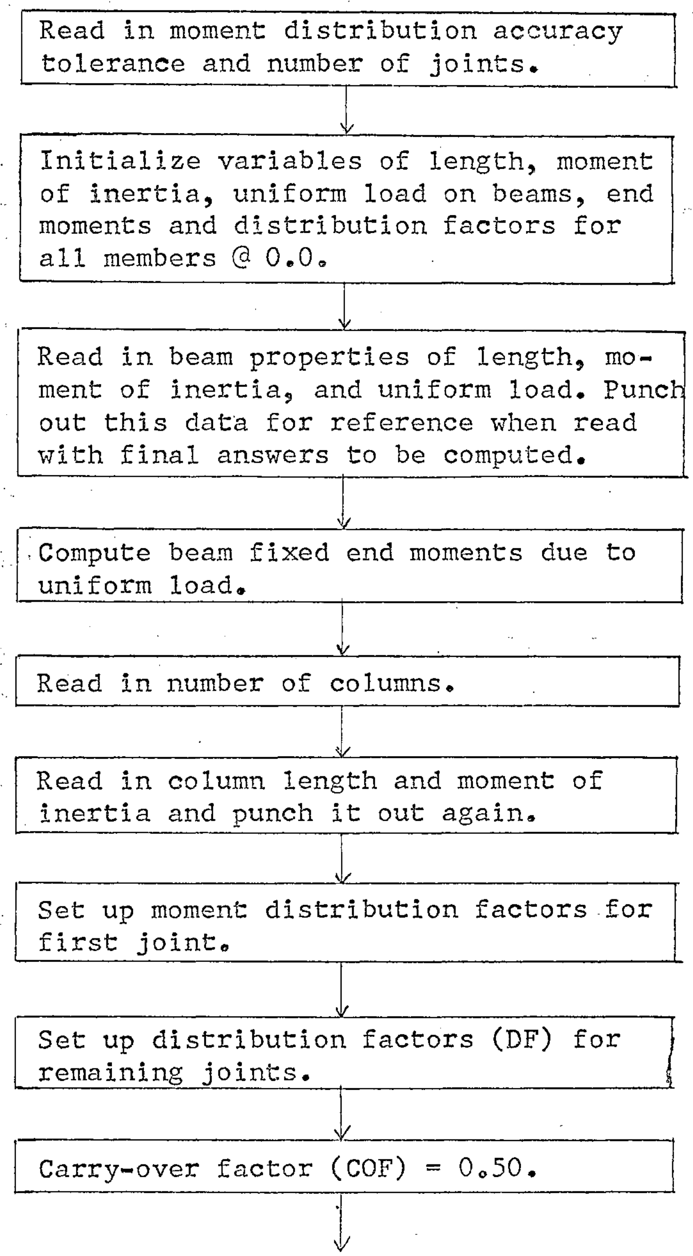


Figure 1.

Flow Chart for Moment Distribution of Single-Story Orthogonal Rigid Frame or Continuous Beam.

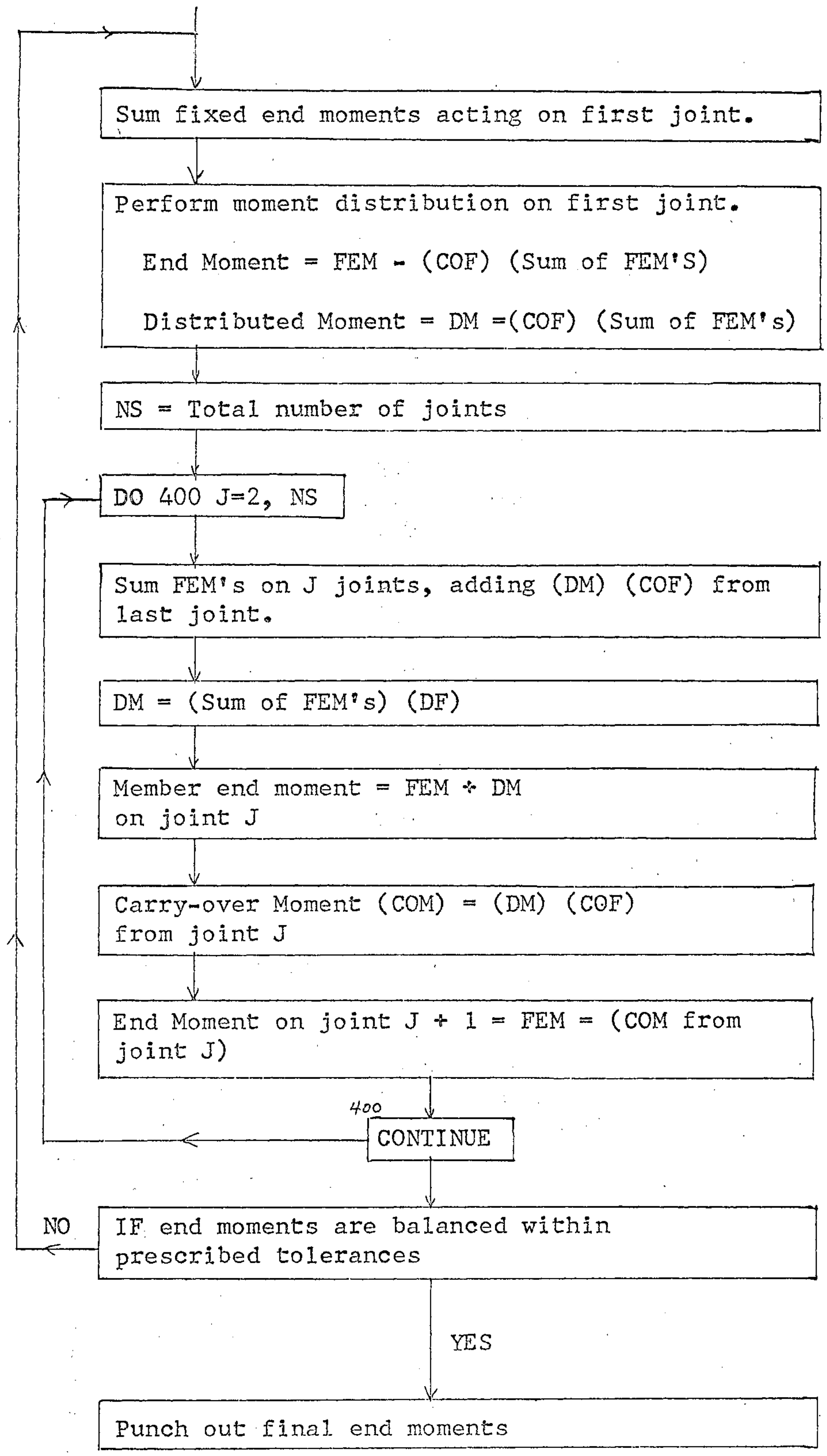


Figure 2.

Continuation of Flow Chart.

TABLE I
INPUT/OUTPUT DATA FOR FRAME

INPUT DATA

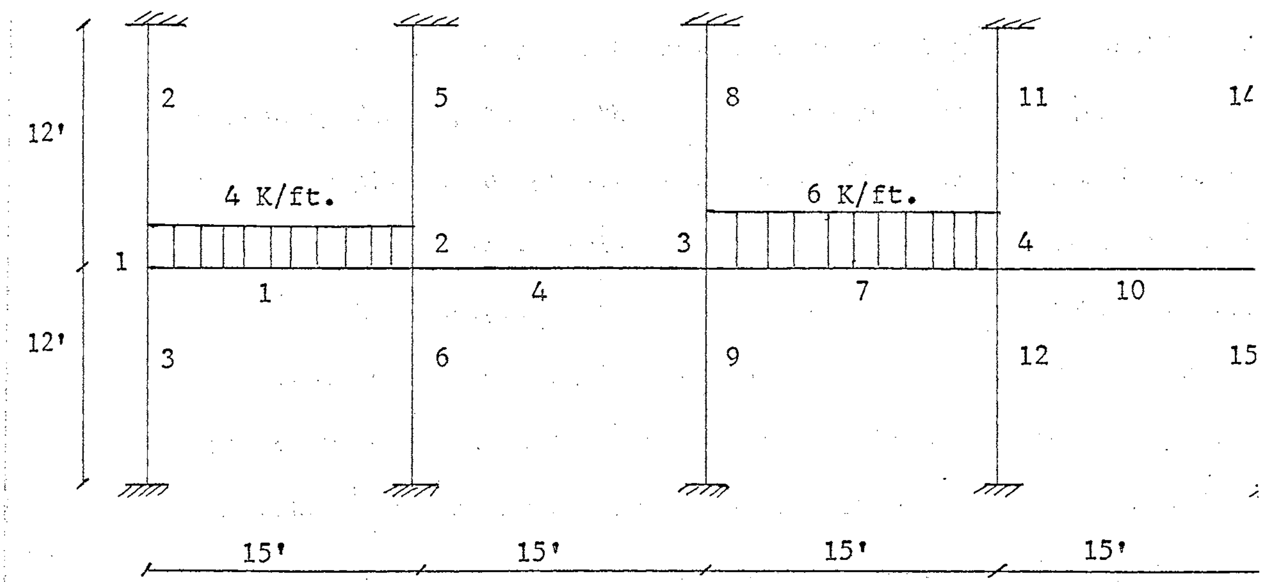
0.001		5	4.0
1	15.0	2.0	0.0
4	15.0	2.0	6.0
7	15.0	2.0	0.0
10	15.0	2.0	
10			
2	12.0	1.0	
3	12.0	1.0	
5	12.0	1.0	
6	12.0	1.0	
8	12.0	1.0	
9	12.0	1.0	
11	12.0	1.0	
12	12.0	1.0	
14	12.0	1.0	
15	12.0	1.0	

OUTPUT DATA

1	15.000000	2.000000	4.000000
4	15.000000	2.000000	0.000000
7	15.000000	2.000000	6.000000
10	15.000000	2.000000	0.000000
10			
2	12.000000	1.000000	
3	12.000000	1.000000	
5	12.000000	1.000000	
6	12.000000	1.000000	
8	12.000000	1.000000	
9	12.000000	1.000000	
11	12.000000	1.000000	
12	12.000000	1.000000	
14	12.000000	1.000000	
15	12.000000	1.000000	

JOINT	MEMBER	BENDING MOMENT
1	1	51.857
1	2	-25.925
1	3	-25.925
2	1	13.215
2	2	22.922
2	3	22.922
2	4	-59.063
3	1	87.235
3	2	-29.326
3	3	-29.326
3	4	-28.583
4	1	38.502
4	2	27.071
4	3	27.071
4	4	-92.645
5	1	0.000
5	2	-6.016
5	3	-6.015
5	4	12.031

Frame with Member Numbers, Joint Numbers and Loads



Moments (ft. - K)

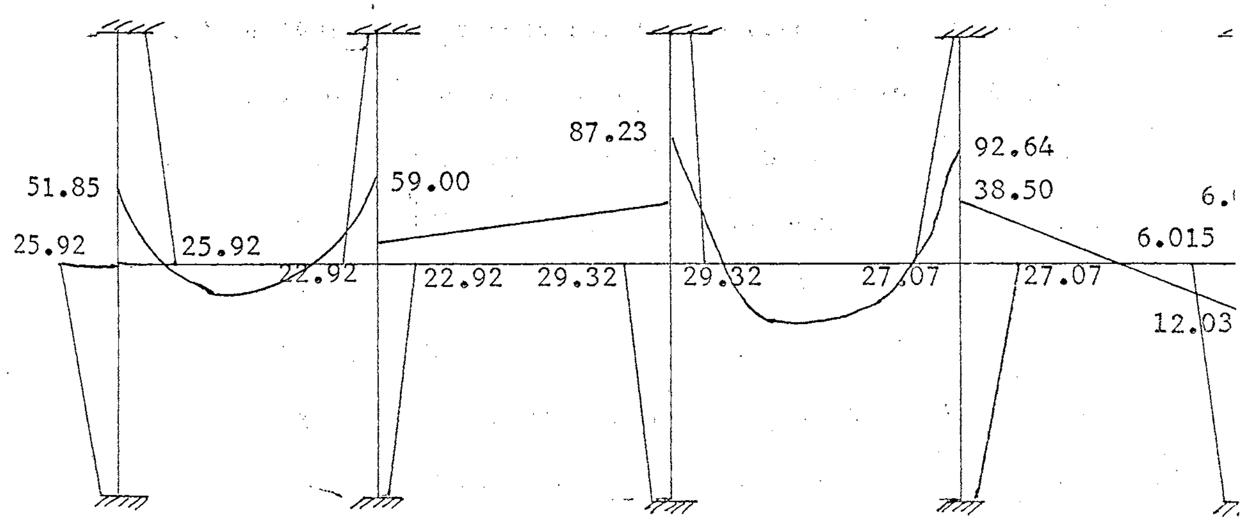


Figure 3.

Sample Frame by Moment Distribution

Analysis of Multi-Story Orthogonal Rigid Frames

This program presents moment distribution in its applicatio to multi-story frames, taking the entire structure at once, rather than one story at a time. The effect of sidesway is considered, and lateral wind loads may be applied. This has a distinct advantage over the prev moment distribution program, because the entire frame may be considered at once, rather than breaking it into parts. This advantage is not witl out a price, however; a larger computer is required than for the previou program. The last program was run on an IBM 1620 computer, with a 20K memory, but this program requires more memory capacity. It might be squeezed onto the 1620, if the size of the dimension statements were reduced. This would largely destroy the advantage of the program, because it would be limited to a very small structure. The program might be broken down into several passes to be run on the 1620, 20K, but this has not been tried. It was run on the IBM 7040, but it would probably run c a 1620, 40K, or an IBM 1130 computer. This has not been tried.

The program, almost in its entirity, has been taken from a publication by Ping-Chun Wang.[1] A few minor modifications have been made. The algorithm for distributing the moments is basically the same as the previous moment distribution program, with the addition of allowance for side sway and lateral loads. The shear in each story due to

[1] Ping-Chun Wang, Numerical and Matrix Methods in Structural Mechanics With Applications to Computers, (John Wiley and Sons, Inc.,) p. 328.

lateral loads is distributed to the columns in proportion to their shea stiffnesses, and from this basic idea the side sway option is built int the program.

The words of the original author best describe the steps in the pr cedure of the program.[2]

> " 1) From the geometry and properties of the frame we find the moment distribution factors of all the members meeting at each joint and shear distribution factors of all the columns in each story. When the columns in each story are of the same height, distribution of unbalanced shear in a story can be accomplished directly by distributing the unbalanced total end moments, which cause unbalanced shears, of all the columns in a story in proportion to the moment of inertia of the columns. The summation of all the end moments of the columns in a story which cause unbalanced shears may be defined as the unbalanced shear moment in that story.
>
> 2) From the loading conditions we compute the fixed end moments of all of the members. The shears in each story are distributed to the columns in that story according to their shear stiffnesses. In turn, the column shears are converted to end moments, $M_{ab} = M_{ba} = -V_{ab} \cdot L_{ab}/2$ for a prismatic column with both ends a and b rigidly connected, or $M_{ab} = -V_{ab} \cdot L_{ab}$ if end b is hinged. Thus the story shears are balanced.
>
> 3) Starting from the lowest floor (it is immaterial which floor we use first), we distribute and carry over the unbalanced moments at all joints.
>
> 4) We compute the unbalanced shear moments induced in adjacent stories after the moment distribution procedure. These moments are represented by the summation of the distributed and carry-over end moments in all the columns of the respective stories.
>
> 5) We distribute the unbalanced shear moment in the story below to the ends of the columns in that

[2]P.C. Wang, Numerical and Matrix Methods in Structural Mechanics With Applications to Computers, (John Wiley and Sons, Inc.), p. 333.

story in proportion to their stiffnesses (moments of inertia in the present case). The unbalanced shear moment in the story above is stored pending the moment distribution of the next higher floor.

6) Thus from floor to floor these procedures are repeated while at each time the unbalanced moments are compared with a predetermined criterion until all the joints and all the stories are balanced in moments and shears to a desired accuracy."

The program is written for an orthogonal frame, with all columns in a given story being of the same length. Members are assumed to be pri matic, with no missing members at any interior bay or story. If a membe is missing, use an imaginary member of zero moment of inertia.

This is an excellent example of how moment distribution can be programmed, in a more useful form than the previous program. Although the running of this program involved little original effort on the part of th author of this report, a report, by its very nature, is a study of the wc of others, and this program was certainly worth presenting, as a typical example of the work that has been done in moment distribution programming

Since this program requires as large a computer as do the slope deflection and stiffness method programs, it should be compared to them in its efficiency of operation. As will be seen in later chapters, the slope deflection and stiffness method programs are more versatile in allo ing frame member configurations other than strictly orthogonal. All of these methods involve an iterative scheme, and the number of iterations, operations, or running time may vary widely between the methods, dependin on the particular problem involved. Moment distribution involves distrib uting the moments to a given accuracy tolerance, and slope deflection or the stiffness method involves solving simultaneous equations to a prescri accuracy. Since computer time is relatively low for small frames, the amount of running time is of little concern. For large problems an eco omy study between these methods might be warranted. Because of the advantages of widely varying member arrangements, the slope deflection an stiffness methods are most commonly used.

A flow chart and sample problem are now presented for this program

Read in number of floors, number of column lines, span of bays, accuracy tolerance for moment distribution, column base condition (pinned or fixed), number of loading conditions, story heights and moments of inertia of columns.

↓

Compute the relative stiffnesses of the first floor and sum these shear stiffnesses.

↓

Compute the shear distribution factor of the first story columns.

↓

Compute the relative stiffnesses of columns framing into each joint.

↓

Read in moments of inertia of beams.

↓

Compute the relative stiffnesses of beams framing into each joint.

↓

Compute the moment distribution factors for all members framing into each joint.

↓

Read in concentrated lateral loads (wind loads) applied to each floor.

↓

Compute the fixed and shear moments of the columns due to wind load.

↓

Read in uniformly distributed loads and concentrated loads applied to each beam.

↓

Compute fixed end moments of beams.

↓

Figure 4.

Flow Chart for Moment Distribution of Multi-Story Frames

Do moment distribution for each floor, and compute induced shear moments in the columns above and below each floor.

Distribute the unbalanced shears for the story below each floor considered, starting down from the top floor.

Compute final shears in the columns.

Print out final column moments and shears.
Print out final beam moments and shears.

Figure 5.

Continuation of Flow Chart for Moment Distribution of Multi-Story Frames

TABLE II
INPUT/OUTPUT DATA FOR SAMPLE MULTI-STORY FRAME

INPUT DATA

3	3	
20.0		
20.0		
0.001	1	1
12.0		
12.0		
36.0		
42.0		
21.6		
14.4		
54.0		
27.6		
24.0		
12.0		
30.0		
16.0		
-4.0		
1.80		
0.60		
-4.0		
1.80	0	
0.60	0	
-4.8		
3.60	0	
1.20	0	

OUTPUT DATA

JOB Bev Edwards 2509-40040

PROBLEM NO. 1

COLUMN MOMENTS AND SHEARS

STORY	COL. LINE	MOM. AT BOT Ft.-KIPS	MOM. AT TOP Ft.-KIPS	SHEAR KIPS
1	1	4.270	34.230	3.208
1	2	-43.949	-57.926	-8.490
1	3	-19.213	-23.012	-3.519
2	1	31.843	31.328	5.264
2	2	-39.453	-44.462	-6.993
2	3	-12.195	-15.062	-2.271

BEAM MOMENTS AND SHEARS

FLOOR	SPAN	MOM. AT LEFT	MOM. AT RIGHT	SHR. AT LEFT	SHR. AT RIGHT
		FT.-KIPS	FT.-KIPS	KIPS	KIPS
2	1	-66.072	146.964	31.955	-40.045
2	2	-49.584	35.208	12.719	-11.281
3	1	-31.327	74.336	15.850	-20.150
3	2	-29.874	15.063	6.741	-5.259

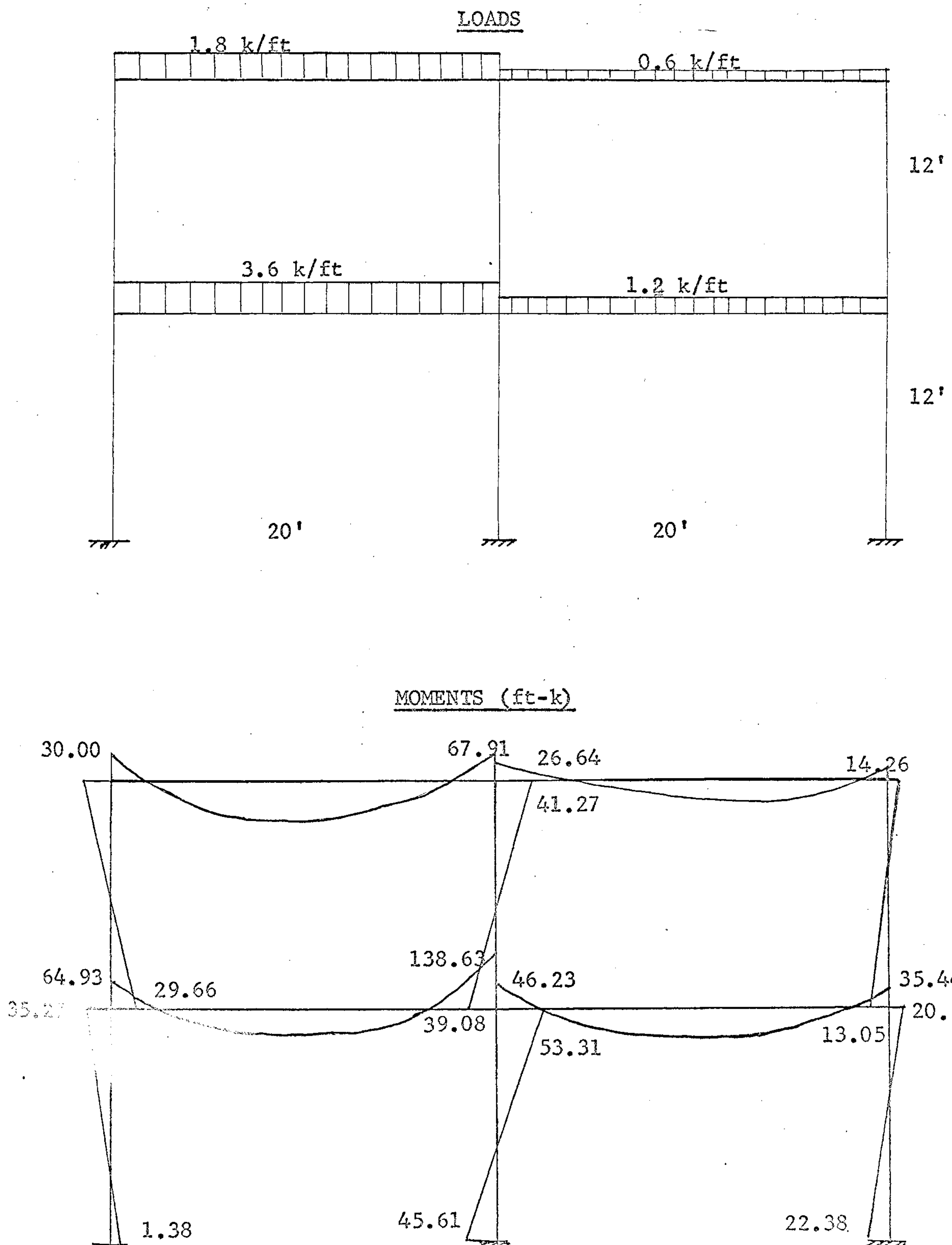


Figure 6

Loads and Final Moments on Frame

SHEAR (kips)

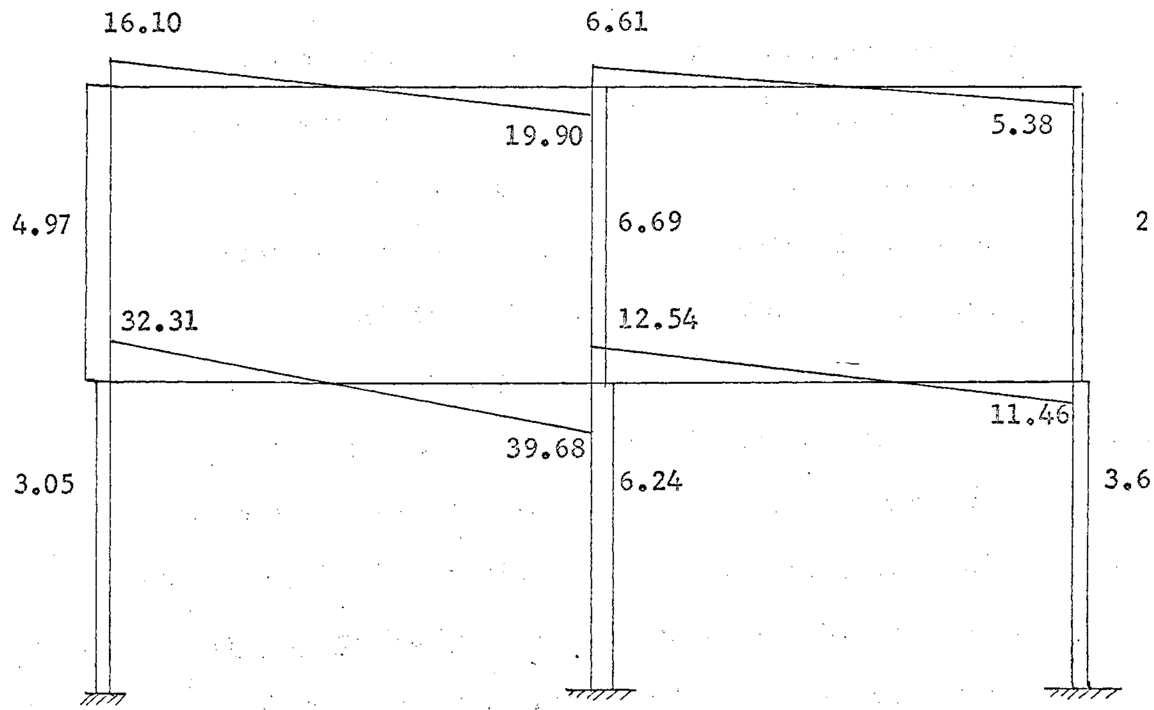


Figure 7.

Shears on Frame

Moment Distribution Factors for Haunched Plate Girders

This program presents an algorithm for determining the moment distribution factors for linearly (as opposed to parabolically) haunched plate girders. The moment distribution factors for haunched beams of rectangular cross section are available in tabular form in "Handbook of Frame Constants", published by the Portland Cement Association. Graphs are available from various texts.

There are no graphs or tables available for the moment distributior factors of I-beams with haunches. At present, these quantities must be calculated by long hand. An algebraic solution to this problem is presented in Moment Distribution, by James M. Gere, (D. Van Nostrand Company, Inc., 1963), pp. 130-133. This algebraic solution has been programmed by the author to give the stiffness factors, carry-over factors, fixed-end moments with a concentrated load. These factors are computed for both ends of the beam, and with the far end fixed or pinned. The stiffness factors calculated by the program are then multiplied by EI/L, where I, the reference moment of inertia, is the moment of inertia of the smaller end or non-haunched end of the beam. In the case of a beam haunched on both ends and uniform in the center section, the reference moment of inertia is that of the center section. E is the modulus of elasticity of the steel, and L is the length of the member.

Fixed-end moments for both ends of the beam are computed for a uni uniform load, and the value punched out by computer is to be multiplie by $w.L.^2$. The fixed-end moment for either end of the beam are computed for a unit concentrated load applied at points 0.1L,0.3L,0.5L, and 0.9

from the left end of the span. These values punched out by the computer are to be multiplied by P L, where P is the value of the concentrated load, and L is the span length.

The input data are the dimensions of the member. Sample dimensions are shown below. All linear dimensions are in inches.

N = Number of Segments to Divide Member into for Analysis
DA = Total Depth of Left End of Beam
DC = Total Depth of Beam at Point of Section Change
DB = Total Depth at Right End of Beam
ZB = Reference Moment of Inertia at Smallest Depth
WF1 = Width of Flange at Left End
TF1 = Thickness of Flange at Left End
TW1 = Thickness of Web at Left End
WF2 = Width of Flange at Right End
TF2 = Thickness of Flange at Right End
TW2 = Thickness of Web at Right End
AFA = Length of Left Haunch/Total Length
BTA = Length of Right Haunch/Total Length

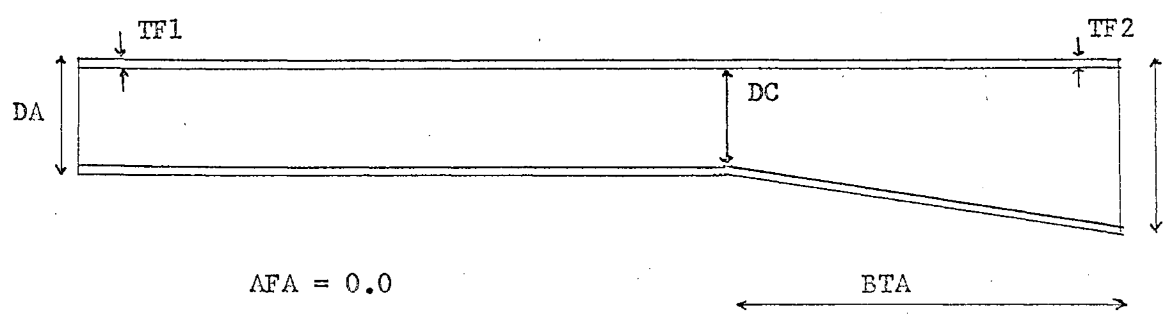


Figure 8.

Typical Haunched Beam

The algorithm for the solution of this problem is not presented in flow chart form, because it merely consists of a series of plug-in equati given in the previous reference. An illustration of some of the possible member shapes that can be solved for is shown next.

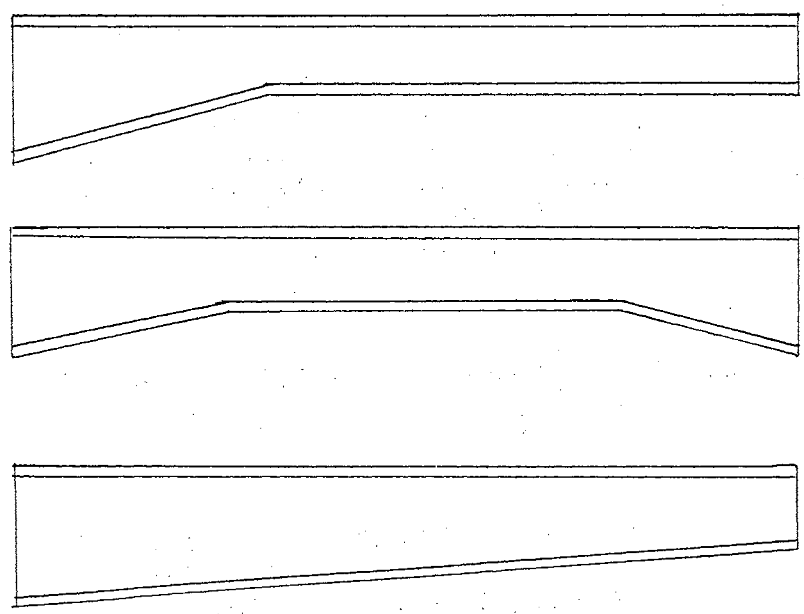


Figure 9.

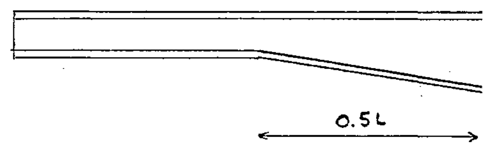
Possible Member Shapes

SAMPLE PROBLEMS[2]

INPUT

30	24.0	24.0	48.0	3620.0
	12.0	1.0	0.50	0.0
	12.0	1.0	0.50	0.50

OUTPUT



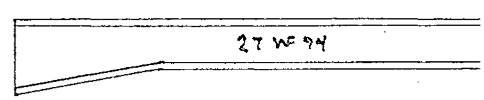
REFERENCE MOMENT OF INERTIA
3619.667000

KF (AB)	KF (BA)	KS (AB)	KS (BA)	
4.720091	9.686037	3.136841	6.437071	
COF (AB)	COF (BA)	UN.L.FM. (AB)	FM (BA)	
.829654	.404298	.065408	-.123705	
K = .1	K = .3	K = .5	K = .7	K = .9
CONC.L. FM (AB)				
.079983	.127817	.090478	.035634	.003643
FM (BA)				
-.015891	-.114841	-.207087	-.201500	-.090759

STIFFNESS FACTORS OBTAINED ARE TO BE MULTIPLIED BY E1/L
F.E.M. FOR UNIFORM LOAD IS MULTIPLIED BY W* (L**2)
F.E.M. FOR CONCENTRATED LOAD IS MULTIPLIED BY P*L

INPUT

30	46.0	26.91	26.91	3267.0
	10.0	1.0	0.50	0.30
	9.990	.747	.49	0.0



[2]These sample problems are taken from Moment Distribution, by James M. Gere, pp. 131, 134.

OUTPUT

REFERENCE MOMENT OF INERTIA
3225.158000

KF (AB)	KF (BA)	KS (AB)	LS (BA)
7.377095	4.592840	4.937462	3.073970
COF (AB)	COF (BA)	UN.L.FM (AB)	FM (BA)
.453750	.728822	.118684	-.068190

	K = .1	K = .3	K = .5	K = 7	K = .9
CONC.L. FM (AB)	.091923	.205868	.193013	.100596	.01331
FM (BA)	-.003583	-.037318	-.097327	-.132745	-.08086

STIFFNESS FACTORS OBTAINED ARE TO BE MULTIPLIED BY El/L
F.E.M. FOR UNIFORM LOAD IS MULTIPLIED BY W* (L**2)
F.E.M. FOR CONCENTRATED LOAD IS MULTIPLIED BY P*L

CHAPTER III

COMPUTER ANALYSIS OF FRAMES BY THE METHOD OF VIRTUAL WORK

Virtual Work Analysis of Orthogonal Rigid Frames

In order to give a full resume of the field of structural prograı ming, samples of as many methods of analysis as possible are shown. : method of virtual work has been programmed.[1] This method of analysis long been in use for manual structural calculations, but it will be ε that it is poorly suited for use on a computer. The difficulty lies preparing the input data for the program. In many cases, it has beer found that a frame could be analyzed quicker by long-hand use of mome distribution than it took to prepare the data for the virtual work pı gram. The data is extremely cumbersome to prepare, and perhaps this will serve to accentuate the advantages of other methods such as slope-deflection and the stiffness method in computer frame analysis.

The virtual work program will solve for the end moments and axial forces in an indeterminate frame of up to twenty degrees of indetermi acy and up to ten loading conditions. There is no practical limit tc the number of members in the structure. The program is set up to tak only concentrated loads applied at the joints. Uniform loads can onl be simulated by a series of concentrated loads applied at dummy joint and this simply is not worth the trouble. For practical frame analys

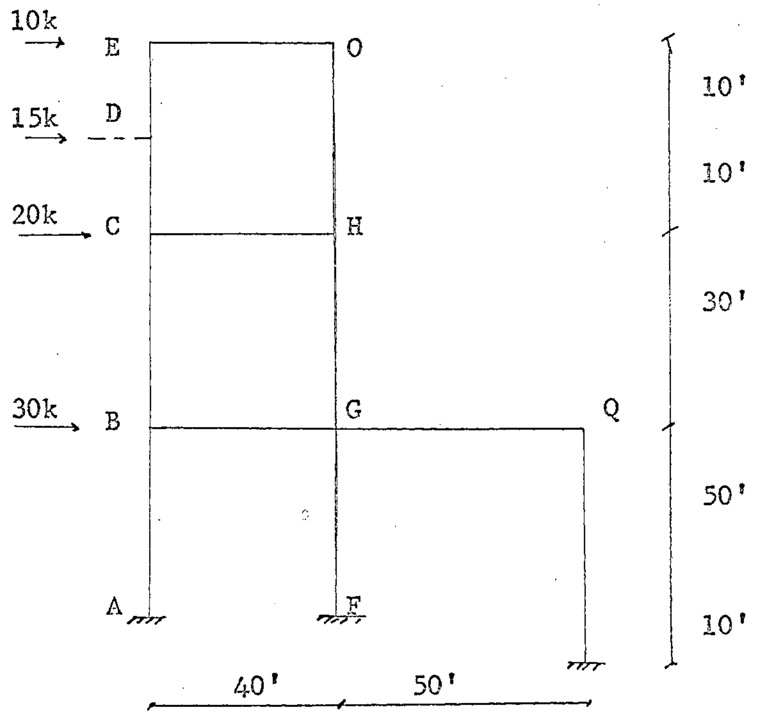
[1] Dravo Corporation, General Virtual Work Analysis of Structures, (I.B.M. Users Group Program Library, Catalog Number 9.2.00.

this narrows down the possible loading conditions to concentrated lateral wind loads only.

The first step in preparing the input data for a plane frame is to cut the structure so that it becomes statically determinate and stable, with redundants applied at the cuts. In a plane rigid frame, there are three redundants at each cut. These are the horizontal and vertical force, and the moment applied to the end of the member cut. For each redundant, a unit force or couple must be applied to the structure in place of the redundant. Expressions are written for the moment at each end of each member and the direct load in place of redundant X(J). Similarly, QM2 (I,J) is the moment at end two of member I, and DIR (I,J) is the direct load in member I due to a unit load in place of redundant X(J). Expressions are not written where the moment or direct load concerned is equal to zero. For a large structure, the tabulation of these Fortran expressions for the values of the two end moments (moment on each end of a member) on each member and the axial load on each member of the structure due to each individual redundant and due to each individual joint load applied to the structure becomes a mountain of tedious work. One sign wrong, in any of the many, many expressions involved, completely destroys the value of this input data. An entire Fortran program has to be written each time a new structure is to be solved. This first program, known as the "Data Preparation Program" has to be written for each new structure. The data preparation program compiles all of the values of the end moments and direct axial loads in the structure caused by the application of the unit values of the redundants and the actual joint loads on the "cut structure". This "Data Preparation Program" prepares the data to be fed to the main virtual work program which does the actual solution of the problem. In the time consumed in writing this data preparation program for each structure, one can usually do the problem long-hand by another method and have more faith in the answer.

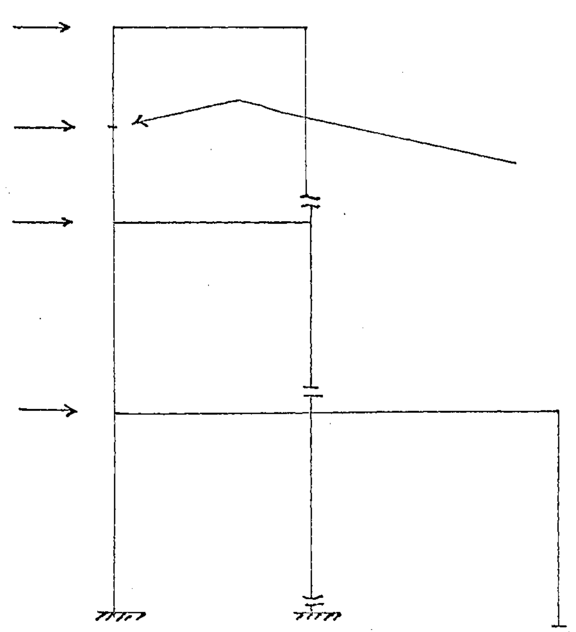
A sample structure is now shown, and it is followed by the data preparation program required to prepare the data for the main virtual work program.

SAMPLE PROBLEM[1]



This building frame will be used to illustrate the procedure.

The frame is cut such that it is determinate and stable.



Dummy joints are located at the points of application of any loads that are not applied at joints.

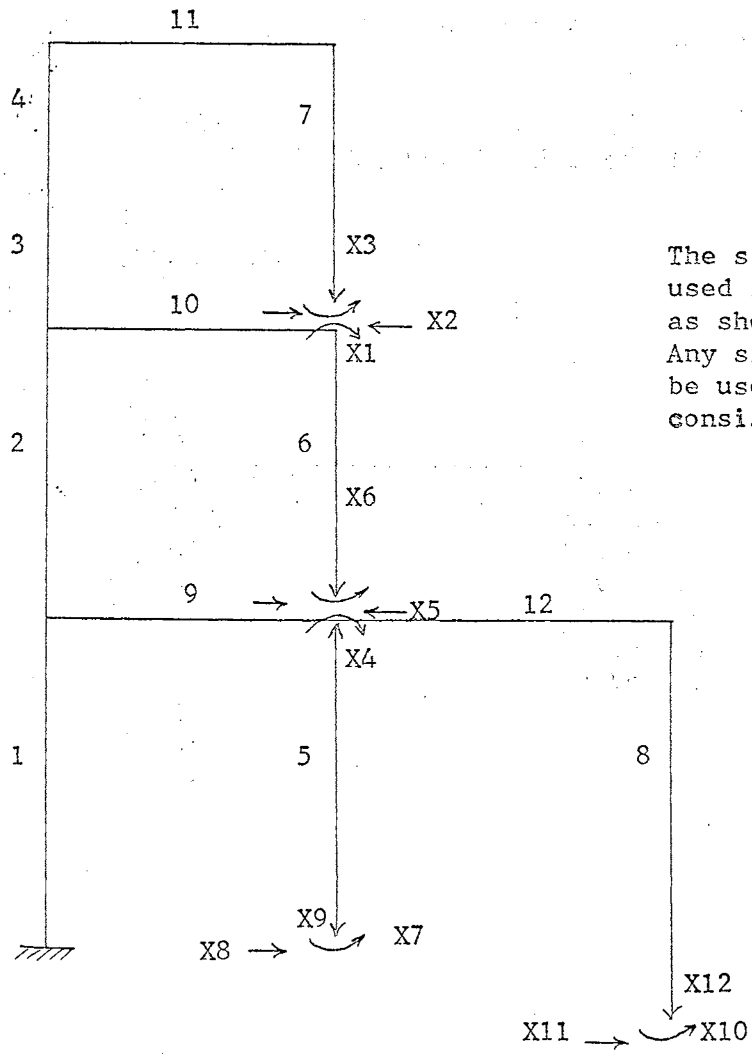
If uniform loads are to be applied, they must be represented as a series of point loads.

Figure 10.

Sample Frame by Method of Virtual Work

[1]I.B.M. Users Group, General Virtual Work Analysis of Structures, p. 8

Redundant reactions are applied at the cut surfaces to bring the structure to equilibrium.



The sign convention used in this problem is as shown on the sketch. Any sign convention may be used as long as it is consistent.

Expressions are written for the moment at each end of each member ar the direct load in each member for unit loads applied in place of each of the redundants.

Figure 11.

Redundants in Frame

```
C                    VIRTUAL WORK FRAME ANALYSIS     PHASE 1
C               THIS PROGRAM PREPARES THE DATA FOR THE MAIN PROGRAM
C               STRESS ANALYSIS FOR THREE STORY BUILDING FRAME
C
C                 EXAMPLE PROBLEM          BY BEV EDWARDS

      DIMENSION QM1 (12,16), QM2 (1a,16), DIR (12,16), QL (12), A (12), QI (1
  201 FORMAT (10F8.3)
  202 FORMAT (3F10.3)
  203 FORMAT (15, 15, 15)
      READ 201, Hi, H2, H3, H4, H5, H6, H7, H8, H9, H10
      READ 203, NM, ND, HL
      DO 1  1=1, NM
    1 READ 202,QL (I), A (I), QI (I)
      NR=ND+NL
      DO 2 I=1, NR
      DO 2 J=1, NM
      QM1 (J,I) = 0.0
      QM2 (J,I) = 0.0
    2 DIR (J,I) = 0.0
      QMI (1,16) = -QL (1) *H1
      QM2 (2,1) = QL (2)
      QM2 (7,1) = -QL (2)
      QM1 (7,1) = QL (2)
      QM2 (1,1) = QL (2)
      DIR (7,1) = 1.0
      QM2 (3,4) = QL (3)
      QM2 (8,4) = -QL (3)
      QM1 (8,4) = QL (3)
      QM2 (7,4) = -QL (3)
      QM1 (7,4) = QL (3)
      QM2 (1,4) = -QL (3)
      DIR (7,4) = 1.0
      DIR (8,4) = 1.0
      QM2 (4,7) = QL (4)
      QM2 (9,7) = QL (4)
      QM1 (9,7) = QL (4)
      QM2 (8,7) = -QL (4)
      QM1 (8,7) = QL (4)
      QM2 (7,7) = -QL (4)
      QM1 (7,7) = QL (4)
      QM2 (1,7) = -QL (4)
      DIR (7,7) = 1.0
      DIR (8,7) = 1.0
      DIR (9,7) = 1.0
      QM2 (5,10) = QL (5)
      QM2 (10,10) = -QL (5)
      QM1 (10,10) = QL (5)
      QM2 (9,10) = -QL (5)
      QM1 (9,10) = QL (5)
```

These expressions for the moment at each end of each member and the direct load in each member for unit loads applied in place of each of the redundants must be punched up for each new probl to be run.

```
QM2 (8,10) = -QL (5)
QM1 (8, 10) = QL (5)
QM2 (7,10) = -QL (5)
QM1 (7,10) = QL (5)
QM2 (1,10) = -QL (5)
DIR (10,10) = 1.0
DIR (9,10) = 1.0
DIR (8,10) = 1.0
DIR (7,10) = 1.0
QM2 (6,13) = QL (6)
QM2 (11,13) = -QL (6)
QM1 (11,13) = QL (6)
QM2 (10,13) = -QL (6)
QM1 (10,13) = QL (6)
QM2 (9,13) = -QL (6)
QM1 (9,13) = QL (6)
QM2 (8,13) = -QL (6)
QM1 (8, 13) = QL (6)
QM2 (7,13) = -QL (6)
QM1 (7,13) = QL (6)
QM2 (1,13) = -QL (6)
DIR (11,13) = 1.0
DIR (10,13) = 1.0
DIR (9,13) = 1.0
DIR (8,13) = 1.0
DIR (7,13) = 1.0
QM1 (7,2) = QL (7)
QM2 (1,2) = -QL (7)
QM1 (1,2) = QL (7)
DIR (2,2) = -1.0
DIR (1,2) = 1.0
QM1 (8,5) = QL (8)
QM2 (7,5) = -QL (8)
QM1 (7,5) = QL (8) + QL (7)
QM2 (1,5) = -QL (8) - QL (7)
QM1 (1,5) = QL (8) + QL (7)
DIR (3,5) = -1.0
DIR (1,5) = 1.0
QM1 (9,8) = QL (9)
QM2 (8,8) = -QL (9)
QM1 (8,8) = QL (9)+ QL (8)
QM2 (7,8) = -QL (9)- QL (8)
QM1 (7,8) = QL (9) + QL (8) + QL (7)
QM2 (1,8) = -QL (9) - QL (8)- QL (7)
QM1 (1,8) = QM1 (7,8)
DIR (4,8) = -1.0
DIR (1,8) = 1.0
QM1 (10,11) = QL (10)
QM2 (9,11) = - QL (10)
QM1 (9,11) = QL (10) + QL (9)
QM2 (8,11) = -QM1 (9,11)
```

```
QM1 (8,11) = QL (10) + QL (9) + QL (8)
QM2 (7,11) = -QM1 (8,11)
QM1 (7,11) = QL (10) + QL (9) + QL (8) + QL (7)
QM2 (1,11) = -QM1 (7,11)
QM1 (1,11) = QM1 (7,11)
DIR (5,11) = -1.0
DIR (1,11) = -1.0
QM1 (11,14) = QL (11)
QM2 (10,14) = -QL (11)
QM1 (10,14) = QL (11) + QL (10)
QM2 (9,14) = -QM1 (10,14)
QM1 (9,14) = QL (11) + QL (10) + QL (9)
QM2 (8,14) = -QM1 (9,14)
QM1 (8,14) = QL (11) + QL (10) + QL (9) + QL (8)
QM2 (7,14) = -QM1 (8,14)
QM1 (7,14) = QL (11) + QL (10) + QL (9) + QL (8) + QL (7)
QM2 (1,14) = -QM1 (7,14)
QM1 (1,14) = QM1 (7,14)
DIR (6,14) = -1.0
DIR (1,14) = 1.0
QM1 (2,3) = 1.0
QM2 (2,3) = -1.0
QM2 (7,3) = 1.0
QM1 (7,3) = -1.0
QM2 (1,3) = 1.0
QM1 (1,3) = -1.0
QM1 (3,6) = 1.0
QM2 (3,6) = -1.0
QM2 (8,6) = 1.0
QM1 (8,6) = -1.0
QM2 (7,6) = 1.0
QM1 (7,6) = -1.0
QM2 (1,6) = 1.0
QM1 (1,6) = -1.0
QM1 (4,9) = 1.0
QM2 (4,9) = -1.0
QM2 (9,9) = 1.0
QM1 (9,9) = -1.0
QM2 (8,9) = 1.0
QM1 (8,9) = -1.0
QM2 (7,9) = 1.0
QM1 (7,9) = -1.0
QM2 (1,9) = 1.0
QM1 (1,9) = -1.0
QM1 (5,12) = 1.0
QM2 (5,12) = -1.0
QM2 (10,12) = 1.0
QM1 (10,12) = -1.0
QM2 (9,12) = 1.0
QM1 (9,12) = -1.0
QM2 (8,12) = 1.0
QM1 (8,12) = -1.0
```

```
      QM2 (7,12) = 1.0
      QM1 (7,12) = -1.0
      QM2 (1,12) = 1.0
      QM1 (1,12) = -1.0
      QM1 (6,15) = 1.0
      QM2 (6,15) = -1.0
      QM2 (11,15) = 1.0
      QM1 (11,15) = -1.0
      QM2 (10,15) = 1.0
      QM1 (10,15) = -1.0
      QM2 (9,15) = 1.0
      QM1 (9,15) = -1.0
      QM2 (8,15) = 1.0
      QM1 (8,15) = -1.0
      QM2 (7,15) = 1.0
      QM1 (7,15) = -1.0
      QM2 (1,15) = 1.0
      QM1 (1,15) = -1.0
```

Expressions for the moments and direct loads due to the unit redundants end here.

```
      PUNCH 203,NM,ND,NL
      DO 10 1=1,NM
      PUNCH 202,QL (I), A (I), QI (I)
      DO 10 J=1,NR
10    PUNCH 202,QM1 (I,J), QM2 (I,J), DIR (I,J)
      STOP
      END
```

TABLE III.
INPUT DATA FOR THE DATA PREPARATION PROGRAM

10000.	15000.	20000.	50000.
12	12	4	
50.0	100.0	10000.	
30.0	75.0	5000.0	
10.0	25.0	2000.0	
10.0	25.0	2000.0	
50.0	90.0	8000.0	
30.0	50.0	3000.0	
20.0	20.0	1500.0	
60.0	60.0	6000.0	
40.0	60.0	6000.0	
40.0	30.0	2000.0	
40.0	25.0	2000.0	
50.0	40.0	3000.0	

TABLE IV
OUTPUT OF THE DATA PREPARATION PROGRAM
INPUT TO THE GENERAL VIRTUAL WORK PROGRAM

12 12	4	
50.000	100.000	10000.000
.000	-30.000	.000
20.000	-20.000	1.000
-1.000	1.000	.000
.000	-10.000	.000
80.000	-80.000	1.000
-1.000	1.000	.000
.000	-10.000	.000
120.000	-120.000	1.000
-1.000	1.000	.000
.000	-50.000	.000
160.000	-160.000	1.000
-1.000	1.000	.000
.000	-30.000	.000
200.000	-200.000	1.000
-1.000	1.000	.000
-500000.0	0.0	0.0
30.000	75.000	5000.000
.000	30.000	.000
.000	.000	-1.000
1.000	-1.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
10.000	25.000	2000.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	10.000	.000
.000	.000	-1.000
1.000	-1.000	.000
.000	.000	.000
.000	.000	.000

.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
10.000	25.000	2000.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	-1.000
1.000	-1.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
50.000	90.000	8000.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	50.000	.000
.000	.000	-1.000
1.000	-1.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
30.000	50.000	3000.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000

.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	30.000	.000
.000	.000	-1.000
1.000	-1.000	.000
.000	.000	.000
20.000	20.000	1500.000
30.000	-30.000	1.000
20.000	.000	.000
-1.000	1.000	.000
10.000	-10.000	1.000
80.000	-60.000	.000
-1.000	1.000	.000
10.000	-10.000	1.000
120.000	-100.000	.000
-1.000	1.000	.000
50.000	-50.000	1.000
160.000	-140.000	.000
-1.000	1.000	.000
30.000	-30.000	1.000
200.000	-180.000	.000
-1.000	1.000	.000
.000	.000	.000
60.000	60.000	6000.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
10.000	-10.000	1.000
60.000	.000	.000
-1.000	1.000	.000
10.000	-10.000	1.000
100.000	-40.000	.000
-1.000	1.000	.000
50.000	-50.000	1.000
140.000	-80.000	.000
-1.000	1.000	.000
30.000	-30.000	1.000
180.000	-120.000	.000
-1.000	1.000	.000
.000	.000	.000
40.000	60.000	6000.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
10.000	-10.000	1.000

40.000	.000	.000
-1.000	1.000	.000
50.000	-50.000	1.000
80.000	-40.000	.000
-1.000	1.000	.000
30.000	-30.000	1.000
120.000	-80.000	.000
-1.000	1.000	.000
.000	.000	.000
40.000	30.000	2000.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
50.000	-50.000	1.000
40.000	.000	.000
-1.000	1.000	.000
30.000	-30.000	1.000
80.000	-40.000	.000
-1.000	1.000	.000
.000	.000	.000
40.000	25.000	2000.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
30.000	-30.000	1.000
40.000	.000	.000
-1.000	1.000	.000
.000	.000	.000
50.000	40.000	3000.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000

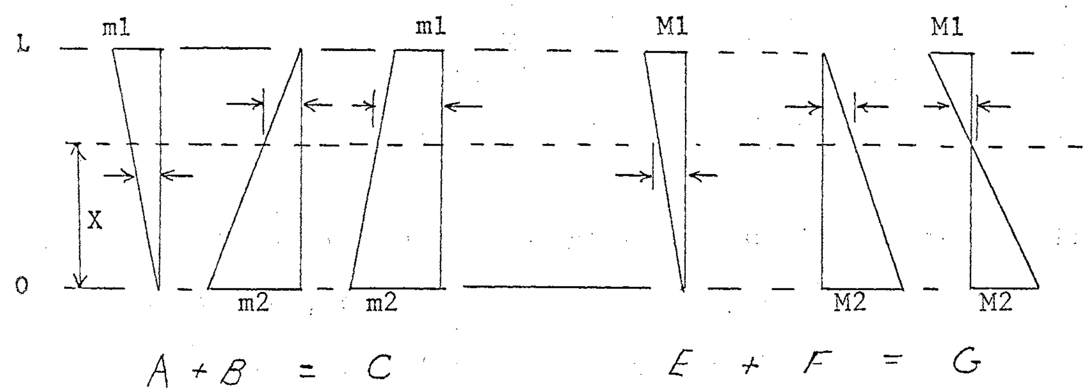
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000
.000	.000	.000

A description of the general virtual work program follows.

The general virtual work equation is employed in the solution for the unknown redundants.

$$D = \Sigma SUL/AE + \Sigma \int Mm \cdot dx/EI \qquad (1)$$

An equation has been derived which satisfies the general equation for the condition that loads are applied only at the joints. Under this condition the moment curve for any member is a straight line.



Refering to the above sketch, the moments at any point, X, are:

$$A = X*m1/L \qquad B = (L-X)*m2/L$$

$$C = A + B = (m1*X + m2*(L-X))/L \qquad (2)$$

In like manner,

$$E = X*m1/L \qquad F = (L-X)*m2/L$$

$$G = E + F = (M1*X + M2(L-X))/L \qquad (3)$$

Substituting C and G for m and M in equation (1)

$$D = \Sigma SUL/AE + \Sigma 1/EI \int_0^L (m1*X + m2*(L-X)*(M1*X + M2*(L-X))/L)dX \qquad (4)$$

Integrating between the limits of 0 and L and simplifying,

$$D = \Sigma SUL/AE + \Sigma L*(m1*(2*M1 + M2) + m2*(2*M2 + M1))/6*EI \qquad (5)$$

The symbols as used in the final Fortran program for equation (5) are:

EQUATION (5)	FORTRAN	ACTUAL MEANING
L	QL	Length
I	QI	Moment of Inertia
A	A	Area
D	D(K,J)	Deflection
S	DIR(K)	Actual Direct Axial Lead
U	DIR(J)	Imaginary Direct Axial Load
M1	QM1(K)	Actual End Moment at End 1
M2	QM2(K)	Actual End Moment at End 2
m1	QM1(J)	Imaginary End Moment at End 1
M2	QM2(J)	Imaginary End Moment at End 2

In this program the following symbols are used to define the limits of the particular problem.

NM = The number of members in the structure

ND = The number of degrees indeterminate

NL = The number of loading conditions

The unit and applied load moments QM1(J), QM2(J), QM1(K). and QM2(K) respectively, and direct loads DIR(J), DIR(K), as computed for the cut structure, in the data preparation program, are fed to the virtual work program member by member along with the properties of the member.

A flow chart for the general virtual work program is presented on the next page.

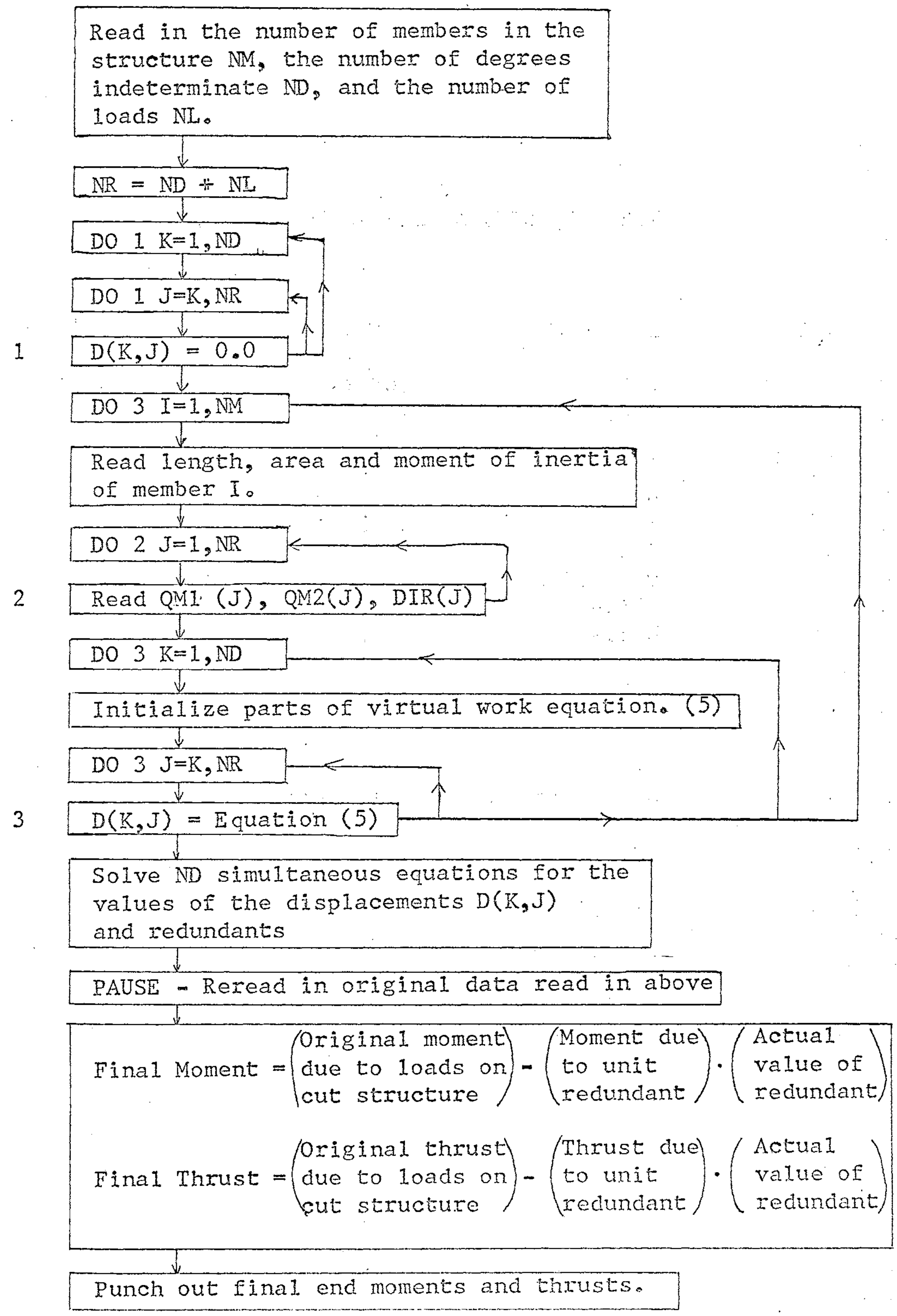


Figure 12.

Flow Chart for General Virtual Work Program

TABLE V
FINAL OUTPUT OF VIRTUAL WORK PROGRAM

MEMBER NUMBER	LOAD NUMBER	END MOM1	END MOM2	DIR LOAD
1	1	-1.705	-.210	.009
1	2	4.447	.559	-.025
1	3	-.107	-.013	0.000
1	4	-227919.700	-29608.560	1188.185
2	1	-.931	-.196	-.006
2	2	2.356	.478	.011
2	3	-.056	-.012	0.000
2	4	-114530.490	-28056.510	-847.476
3	1	-.554	-.121	-.009
3	2	1.053	-.670	-.038
3	3	-.020	.011	0.000
3	4	-1134.341	7914.061	-232.600
4	1	-2.369	-2.605	-.058
4	2	4.207	-2.804	-.023
4	3	-.086	.051	.002
4	4	11145.621	11173.686	109.941
5	1	3.122	-24.428	.064
5	2	21.506	-31.134	1.075
5	3	-.519	.753	-.003
5	4	-110308.530	-18675.230	-218.049
6	1	0.0000	30.000	0.000
6	2	0.0000	0.000	-1.000
6	3	1.0000	-1.000	0.000
6	4	0.0000	0.000	0.000
7	1	.210	-.020	-.028
7	2	-.559	.053	.080
7	3	.013	-.001	-.001
7	4	29608.560	-5844.870	-4422.672
8	1	.216	-.007	.008
8	2	-.531	-.267	-.013
8	3	.013	.003	0.000
8	4	33901.370	-13458.830	330.227
9	1	.128	-.373	.076
9	2	.937	-3.028	-.052
9	3	-.015	.064	.001
9	4	5544.770	-1220.440	-347.744
10	1	2.979	-5.571	.573
10	2	5.832	-8.865	-.192
10	3	-.116	.246	.004
10	4	-9953.240	18675.240	-2579.675

11	1	30.000	-30.000	1.00
11	2	40.000	0.000	0.00
11	3	-1.000	1.000	0.00
11	4	0.000	0.000	0.00
12	1	0.000	0.000	0.00
12	2	0.000	0.000	0.00
12	3	0.000	0.000	0.00
12	4	0.000	0.000	0.00

CHAPTER IV

COMPUTER ANALYSIS OF FRAMES BY THE SLOPE-DEFLECTION METHOD

Slope-Deflection Analysis of Orthogonal Rigid Frames

The I.B.M. Users Group presented one of the first programs available for the analysis of highly indeterminate multi-story rigid frames.[1] This program, or similar versions of it, has been used successfully for several years by engineers in evaluating the moments and shears present in large multi-story frames. If it is used to its fullest advantage, it should replace the cantilever and portal methods of approximate analysis. It should be listed as one of the outstanding advantages of computer analysis over manual methods.

The slope-deflection method has been programmed to determine member end moments and reactions for a statically indeterminate orthogonal plane frame. It may be used to determine the moments and reactions for a number of different loading conditions acting on the same frame.

> "The program is written in two passes. Pass I evaluates and inverts a stiffness matrix for the structure. Pass II computes end moments and reactions by multiplication of load vectors for each loading condition by the inverted structure matrix from Pass I. Axial shortening in the members is assumed to have a negligible effect, as is shear deflection. Only bending deflections are considered."[1]

[1] Kenneth Marvin Richmond, Structural Frame Analysis Program, (IBM Company, Ltd., 1445 West Georgia Street, Vancouver 5, B.C.)

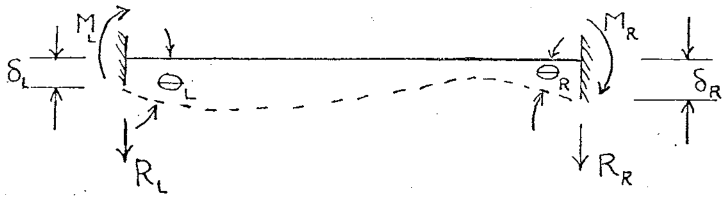
The original version of the program was written in Fortran II, for use on the IBM 1620, with a 40K memory. The author of this report has rewritten the program in Fortran IV, to be run on the IBM 7040 computer. This enables the program to be used for frames of higher indeterminacy, with the increased memory capacity of the 7040. At present, the program is set up to take a structure of up to 50 members, and up to 40 unknown translations or rotations of the joints. For a structure with 40 unknown joint displacements, this would mean that a 40 x 40 stiffness matrix would be inverted. With the large memory of the 7040, the allowable size of the matrix could probably be increased to a maximum of about 150 x 150, for this program. Larger matrices can be solved using algerithms other than the one employed in this program, but for most structural problems encountered, the program is adequate in its present form. A problem with a 40 x 40 stiffness matrix could probably be handled by an IBM 1130 computer.

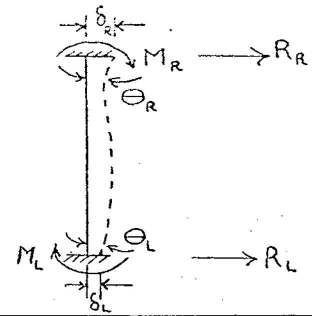
> "The program reads cards describing each member and evaluates a stiffness matrix for the individual members by computing the coefficients of the four slope-deflection equations and summing up the elements of the individual member matrices. The resulting structure stiffness matrix is inverted to give a flexibility matrix for the structure. For each loading case a load vector is made up representing the unbalanced bending moments at each joint and the unbalanced bending reactions for each lateral floor deflection or unknown vertical deflection. The load vector is premultiplied by the flexibility matrix to give a deflection vector made up of all the unknown rotations and deflections at the joints. Finally, the resulting end moments and reactions for all members are determined from the deflections and initial fixed-end moments and reactions. Input for the load cases is the fixed-end moments and

reactions for each loaded member and output is final moments and reactions for all members. A number of loading conditions can be solved for each frame."[2]

Members can be vertical or horizontal, but no provision has been made to handle sloping members. Coefficients are computed by the program for prismatic members built in at both ends or pinned at either end. For non-prismatic members, there is a provision to enter the coefficients of the 4 by 4 stiffness matrix for the member on data cards. There are four types of members, the fourth of which is the non-prismatic type. The first three types and their stiffness matrices are shown on the next page. It should be noted that the matrices are symmetric for all four types of members.

[2] Kenneth Marvin Richmond, Structural Frame Analysis Program, (IBM Company Ltd., 1445 West Georgia Street, Vancouver 5, B.C.)





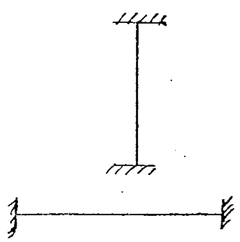
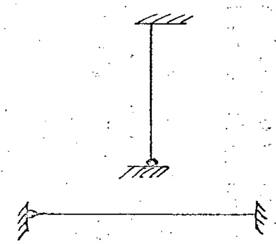
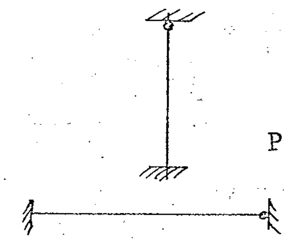
TYPE 1				TYPE 2				TYPE 3			
				Pin 				 Pin			
4EI/L	6EI/L^2	2EI/L	-6EI/L^2	0	0	0	0	3EI/L	3EI/L^2	0	-3EI/L^2
6EI/L^2	12EI/L^3	6EI/L^2	12EI/L^3	0	3EI/L^3	3EI/L^2	-3EI/L^3	3EI/L^2	3EI/L^3	0	-3EI/L^3
2EI/L	6EI/L^2	4EI/L	-6EI/L^3	0	3EI/L^2	3EI/L	-3EI/L^2	0	0	0	0
-6EI/L^2	-12EI/L^3	-6EI/L^2	12EI/L^2	0	-3EI/L^3	-3EI/L^2	-3EI/L^3	-3EI/L^2	-3EI/L^3	0	3EI/L^3

Figure 13.

Stiffness Matrix for Slope Deflection

"Each member in the structure must be defined by a member description card giving its member number for reference purposes, its stiffness EI, length, and member type (1-4), along with some data identifying the unknown end deflections. If the member is a type 1, 2, or 3 member, the values of EI and L given are used to compute a stiffness matrix for the member in accordance with the table just given. If the member is type 4, then the 4 cards immediately following the member description card must contain the elements of the stiffness matrix, for the member, arranged as shown for type 1, 2, or 3 member. These four cards are included only for type 4 members. The sign convention used is as follows:

1) Rotations and bending moments are positive in the clockwise direction.

2) Deflections and forces are positive when acting down or to the right.

When the data is to be prepared, a sketch should be made of the structure, and a number is assigned to each member, from 1 to NM. It is not necessary to follow a particular order in assigning numbers to the members. Then each unknown joint rotation or deflection must be assigned a number. Again, all numbers from 1 to NDF, the number of degrees of indeterminacy of the structure, must be used. Where an entire floor must deflect laterally as a unit, the same horizontal deflection number will apply to every joint at that floor level."[3]

A flow chart for the program is given next.

[3] Kenneth Marvin Richmond, Structural Frame Analysis Program, (IBM Company Ltd., 1445 West Georgia Street, Vancouver 5, B.C.)

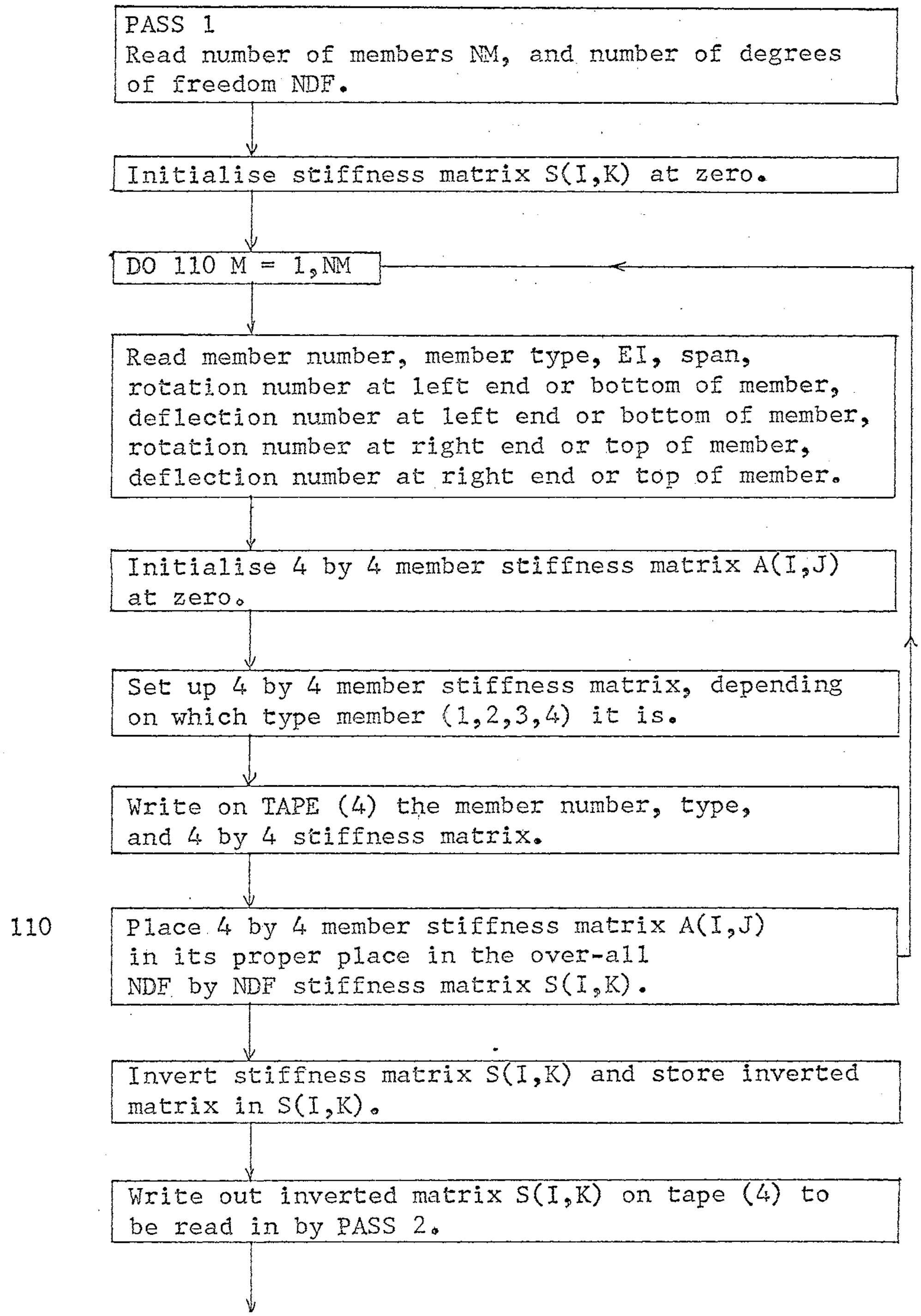


Figure 14.

Flow Chart for Pass 1 of Slope-Deflection Method

PASS 2
Read in member data and inverted stiffness matrix written on tape (4) in PASS 1.

↓

Read in loading conditions.

↓

Multiply load vector by inverted matrix.

↓

Compute moments and shears.

↓

Punch out final end moments and shears for each member.

Figure 15.

Flow Chart for Pass 2 of Slope-Deflection Method

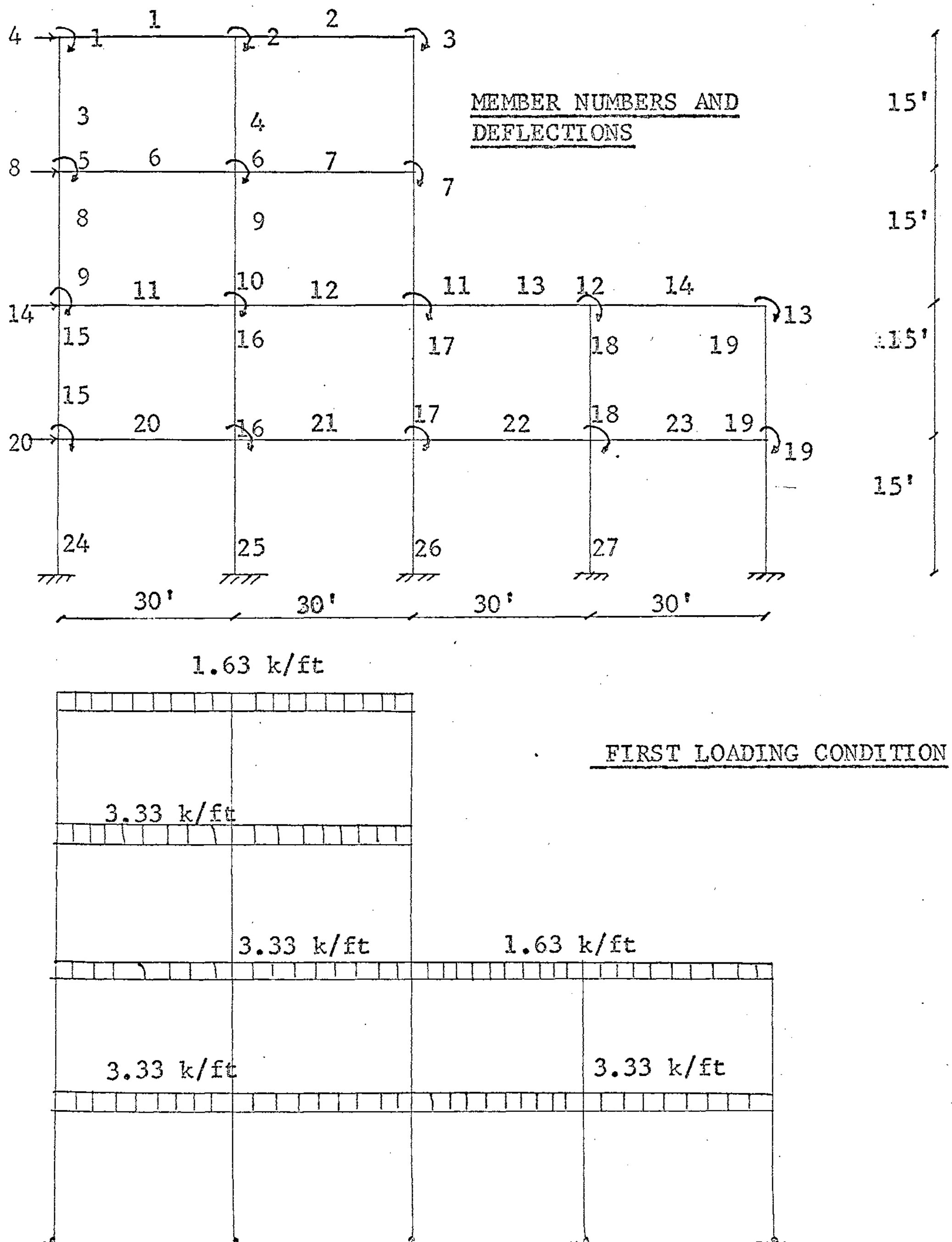
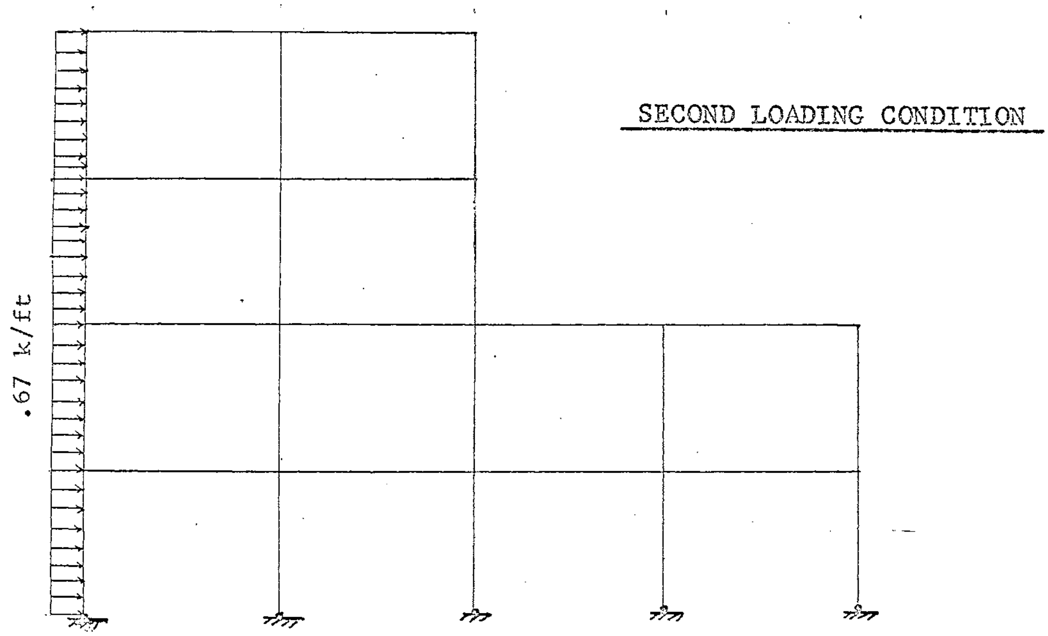


Figure 16.

Sample Problem by Slope-Deflection Method.



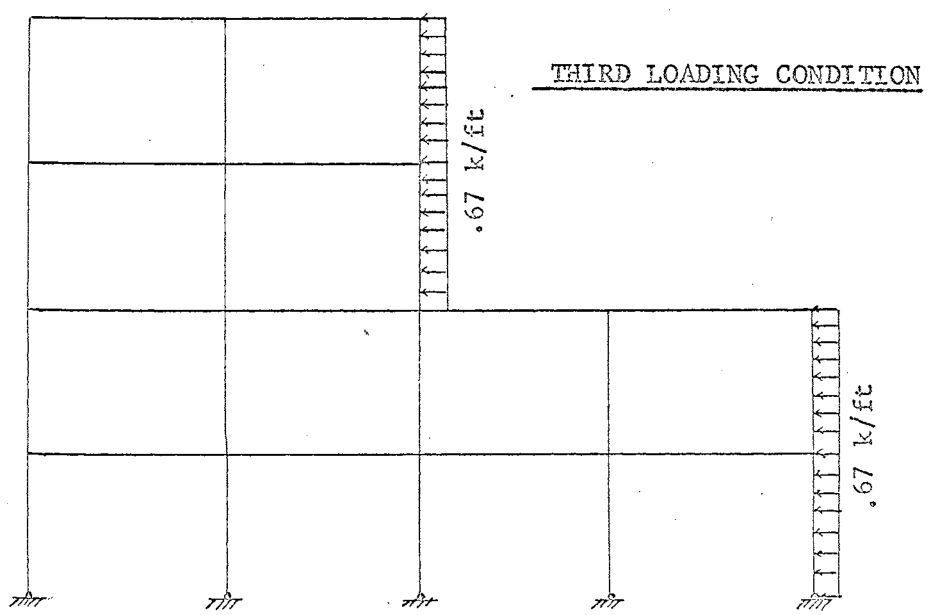


Figure 17.

Loading Conditions

TABLE VI
INPUT DATA FOR PASS 1 -- MEMBER DATA

2820

1	1	11.58	30.0	1	0	2	0
2	1	11.58	30.0	2	0	3	0
3	1	2.28	15.0	5	8	1	4
4	1	2.28	15.0	6	8	2	4
5	1	2.28	15.0	7	8	3	4
6	1	34.2	30.0	5	0	6	0
7	1	34.2	30.0	6	0	7	0
8	1	3.60	15.0	9	14	5	8
9	1	3.03	15.0	10	14	6	8
10	1	3.60	15.0	11	14	7	8
11	1	38.8	30.0	9	0	10	0
12	1	34.2	30.0	10	0	11	0
13	1	11.58	30.0	11	0	12	0
14	1	11.58	30.0	12	0	13	0
15	1	4.42	15.0	15	20	9	14
16	1	6.90	15.0	16	20	10	144
17	1	3.60	15.0	17	20	11	14
18	1	2.28	15.0	18	20	12	14
19	1	3.03	15.0	19	20	13	14
20	1	52.1	30.0	15	0	16	0
21	1	47.2	30.0	16	0	17	0
22	1	34.2	30.0	17	0	18	0
23	1	34.2	30.0	18	0	19	0
24	2	12.40	15.0	0	0	15	20
25	2	19.90	15.0	0	0	16	20
26	2	12.40	15.0	0	0	17	20
27	2	3.95	15.0	0	0	18	20
28	2	3.60	15.0	0	0	19	20

TABLE VII

INPUT DATA FOR PASS 2 - LOAD DATA

12	1			
1	122.	24.4	-122.	24.4
2	122.	24.4	-122.0	24.4
6	250.0	50.0	-250.0	50.0
7	250.0	50.0	-250.0	50.0
11	250.0	50.0	-250.0	50.0
12	250.0	50.0	-250.0	50.0
13	122.0	24.4	-122.0	24.4
14	122.0	24.4	-122.0	24.4
20	250.0	50.0	-250.0	50.0
21	250.0	50.0	-250.0	50.0
22	250.0	50.0	-250.0	50.0
23	250.0	50.0	-250.0	50.0
4	2			
3	16.66	5.0	-16.66	5.0
8	16.66	5.0	-16.66	5.0
15	16.66	5.0	-16.66	5.0
24	0.0	3.75	-25.0	6.25
4	3			
5	-16.66	-5.0	16.66	-5.0
10	-16.66	-5.0	16.66	-5.0
19	-16.66	-5.0	16.66	-5.0
28	0.0	-3.75	25.0	-6.75

TABLE VIII

FINAL OUTPUT OF PASS 2 - END MOMENTS AND REACTIONS

MN	BML	RL	BMR	RR	LOADING 1
1	42.4	20.42	-161.7	28.38	
2	161.9	28.38	-42.5	20.42	
3	-37.7	-5.34	-42.4	5.34	
4	-0.3	-0.03	-0.1	0.03	
5	38.0	5.37	42.5	-5.37	
6	83.8	41.63	-335.1	58.37	
7	331.3	58.26	-83.5	41.74	
8	-43.8	-6.00	-46.2	6.00	
9	6.0	0.67	4.1	-0.67	
10	34.4	5.33	45.5	-5.33	
11	98.4	42.94	-310.3	57.06	
12	299.6	54.05	-178.2	45.95	
13	133.9	24.53	-130.0	24.27	
14	142.0	27.33	-54.2	21.47	
15	-49.8	-6.96	-54.6	6.96	
16	4.7	0.63	4.7	-0.63	
17	3.2	0.87	9.9	-0.87	
18	-10.9	-1.53	-12.0	1.53	
19	50.7	6.99	54.2	-6.99	
20	102.9	43.34	-302.7	56.66	
21	275.0	51.24	-237.8	48.76	
22	230.3	48.05	-288.9	51.95	
23	305.1	57.45	-81.6	42.55	
24	-0.0	-2.54	-53.0	3.54	
25	-0.0	1.54	23.1	-1.54	
26	-0.0	0.29	4.3	-0.29	
27	-0.0	-0.35	-5.3	0.35	
28	-0.0	2.06	30.9	-2.06	

LOADING 2

MN	BML	RL	BMR	RR
1	1.4	-0.08	-3.9	0.08
2	-9.9	-0.68	-10.6	0.68
3	28.1	6.78	-1.4	3.22
4	14.7	1.90	13.8	-1.90
5	9.2	1.32	10.6	-1.32
6	-47.7	-2.51	-27.5	2.51
7	-27.0	-2.46	-46.7	2.46
8	52.4	9.80	19.6	0.20
9	37.9	5.18	39.8	-5.18
10	37.7	5.01	37.5	-5.01
11	-72.9	-4.40	-59.2	4.40
12	-47.7	-3.43	-55.2	3.43
13	-16.6	-0.95	-12.0	0.95
14	-15.7	-1.33	-24.1	1.33
15	51.7	9.81	20.5	0.19
16	61.9	8.72	68.9	-8.72
17	33.3	4.49	34.1	-4.49
18	27.9	3.71	27.7	-3.71
19	24.8	3.26	24.1	-3.26
20	-149.5	-9.82	-145.1	9.82
21	-119.0	-7.65	-110.6	7.65
22	-55.3	-3.06	-36.6	3.06
23	-40.0	-3.41	-62.2	3.41
24	-0.0	11.94	97.8	-1.94
25	-0.0	13.48	202.2	-13.48
26	-0.0	8.84	132.6	-9.84
27	-0.0	3.25	48.7	-3.25
28	-0.0	2.49	37.4	-2.49

MN	BML	RL	BMR	RR	LOADING 3
1	10.7	0.69	9.9	-0.69	
2	3.9	0.08	-1.4	-0.08	
3	-9.2	-1.33	-10.7	1.33	
4	-14.7	-1.90	-13.8	1.90	
5	-28.0	-6.78	1.4	-3.22	
6	46.3	2.45	27.1	-2.45	
7	27.9	2.53	48.0	-2.53	
8	-36.3	-4.89	-37.0	4.89	
9	-38.6	-5.26	-40.3	5.26	
10	-52.7	-9.85	-20.0	-0.15	
11	72.2	4.31	57.1	-4.31	
12	50.4	3.80	63.7	-3.80	
13	20.6	1.19	15.2	-1.19	
14	11.6	0.83	13.4	-0.83	
15	-33.3	-4.61	-35.9	4.61	
16	-61.4	-8.69	-68.8	8.69	
17	-31.8	-4.22	-31.6	4.22	
18	-27.2	-3.59	-26.7	3.59	
19	-44.8	-8.88	-13.4	-1.12	
20	156.0	10.14	148.2	-10.14	
21	119.0	7.66	110.7	-7.66	
22	55.9	3.11	37.6	-3.11	
23	38.9	3.25	58.5	-3.25	
24	0.0	-8.18	-122.7	8.18	
25	0.0	-13.72	-205.8	13.72	
26	0.0	-8.98	-134.8	8.98	
27	0.0	-3.29	-49.3	3.29	
28	0.0	-6.33	-13.7	-4.17	

CHAPTER V

COMPUTER ANALYSIS OF FRAMES AND TRUSSES BY THE STIFFNESS METHOD

The stiffness method is probably the method of structural analysis which is best suited for use on a computer. The program just presented for the slope-deflection method's use of the stiffness matrix is a program of somewhat limited capability. It is excellent for that special type of rigid frame whose member arrangement is orthogonal, but it lacks the flexibility to be used for frames like the gable, where the member arrangement is not orthogonal. In this chapter the use of the stiffness matrix will be expanded to operate on members whose axis do not necessarily lie on the X or Y axis. In their most general form, plane trusses and plane frames may have members lying at any angle with respect to the X, Y. coordinates. Likewise, space trusses and space frames may have members skewed with respect to the X, Y, Z coordinates. As will be seen later, the stiffness matrices can be modified by pre- and post-multiplication by rotation matrices to account for the orientation of members away from the main coordinate system.

The programming of any method of analysis requires a formalized pattern of logical decisions, and the stiffness method is no exception. A computer program for the analysis of a structure by the stiffness method is conveniently divided into several phases, regardless of whether it is for the analysis of a continuous beam, a grid, a plane truss, a plane frame, a space truss or a space frame. The basic approach is explained by James Gere and William Weaver, Jr., in their text, Analysis of Framed Structures.[1]

"(1) Assembly of Structure Data. Information pertaining to the structure itself must be assembled and recorded. This information includes the number of members, the number of joints, the number of degrees of freedom, and the elastic properties of the material. The locations of the joints of the structure are specified by means of geometric coordinates. In addition, the section properties of each member in the structure must be given. Finally, the conditions of restraint at the supports of the structure must be identified. In computer programming, all such information is coded in some convenient way.

(2) Generation and Inversion of Stiffness Matrix. The stiffness matrix is an inherent property of the structure and is based upon the structure data only. In computer programming, it is convenient to obtain the joint stiffness matrices. This involves generalizing the joint stiffness matrix from one that is related only to the degrees of freedom in the structure to one that is related to all possible joint displacements, including support displacements. This generalized stiffness matrix is called the 'over-all joint stiffness matrix'.

(3) Assembly of Load Data. All loads acting on the structure must be specified in a manner which is suitable for computer programming. Both joint loads and member loads must be given. The former may be handled directly, but the latter are handled indirectly by supplying as data the fixed-end actions caused by the loads on the members.

(4) Generation of Vectors Associated with Loads. The fixed-end actions due to loads on members may be converted to 'equivalent joint loads'. These equivalent joint loads may then be added to the actual joint loads to produce a problem in which the structure is imagined to be loaded at the joints only.

[1] James M. Gere and William Weaver, Jr., Analysis of Framed Structures, (D. Van Nostrand Company, Inc., 1965)

(5) Calculation of Results. In the final phase of the analysis all of the joint displacements, reactions, and member end-actions are computed. One performs the calculation of member end-actions member by member, instead of considering the structure as a whole. Such calculations require the use of member stiffness matrices.

It should be noted that there are many possible variations in organizing the stiffness method for programming. The phases of the analysis listed above constitute an orderly approach which has certain essential features that are advantageous when dealing with large, complicated frameworks."[2]

Four programs utilizing the stiffness method are presented in this chapter. As outlined above, the basic organization of the stiffness method approach is the same for any problem. The stiffness coefficients and restraint conditions for a plane truss are different from those of a space frame, but the general approach is the same. Many of the Fortran statements in the plane truss, plane frame, space truss, and space frame programs are interchangeable. The identically same algerithm has been used to invert the stiffness matrix S in the programs for slope-deflection analysis of plane trusses, plane frames, and space frames. The central figure of all stiffness method problems is the stiffness matrix itself, and its inversion.

A flow chart for the inversion of the stiffness matrix S is presented on the next page.

[2] James M. Gere and William Weaver, Jr., Analysis of Framed Structures, (D. Van Nostrand Company, Inc., 1965) pp. 191-192.

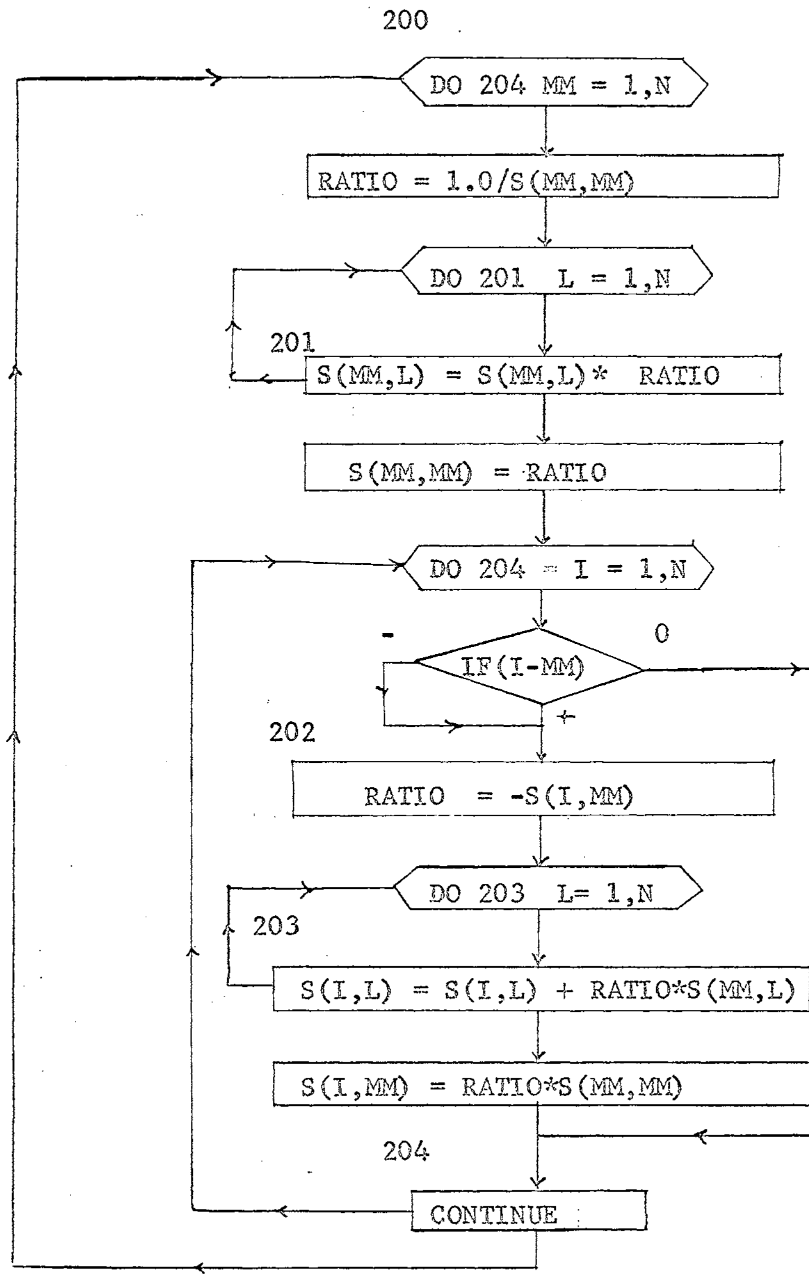


Figure 18.

Invert Stiffness Matrix S and Store Inverted Matrix in S.

Plane Truss Analysis by the Stiffness Method

The analysis of a plane truss by the stiffness method follows the pattern previously outlined for any stiffness method solution. Trusses differ from other types of structures in the arrangement of their member stiffness matrix and in their degrees of freedom. All members are considered to be pinned-end, with two degrees of freedom possible in each joint (X and Y translation), unless retrained by a support. The size of the over-all joint stiffness matrix is determined by the number of joints in the structure. If there are ten joints in a truss, the over-all joint stiffness matrix will be 20 by 20, for the 20 possible degrees of freedom (two/joint). Some of these degrees of freedom or displacements will undoubtedly be removed by the restraint action of the supports. If two supports remove two possible displacements per support, for a total of four restraints, the final matrix involving the unknown displacements will be 16 by 16. It is this 16 by 16 part of the over-all joint stiffness matrix which is partitioned off from the over-all matrix and inverted to find the unknown displacements.

If the analysis of a plane truss, as in the case of any other type of framed structure, it is convenient to generate the joint stiffness matrix S, by assessing the contributions from the member stiffnesses. The individual contributions of the 4 by 4 member stiffness matrices for each member are put in their proper place in the joint stiffness matrix according to the unknown displacements involved, just as the simultaneous equations for a slope-deflection solution are set up. The member stiffness matrix for each member of the truss is a 4 by 4 matrix, because there are four possible displacements for each member of a plane truss. These displacements are the X and Y displacements of each end of the member. Although it is assumed that the reader is familiar with the manual calculations involved in doing a truss solution by long-hand, a study of the arrangement of the matrices is important in understanding the computer solution. The plane truss member stiffness matrix for a member with respect to its own member axes (as opposed to the axes oriented to the structure) is shown here.

$$S_m = EA/L \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

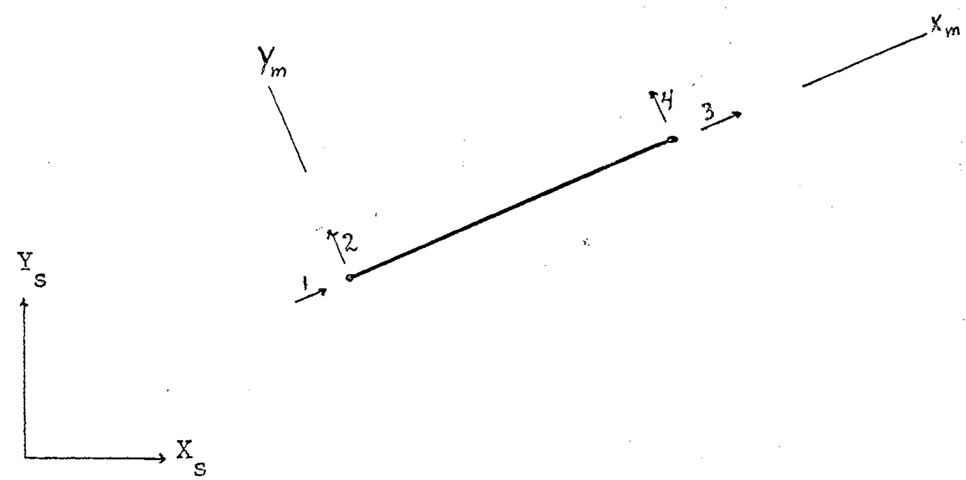


Figure 19.

Typical Plane Truss Member

As shown above, member i, with ends j and k, is at an angle with the structure oriented axes X_s and Y_s . The member has four possible end displacements (1,2,3,4), along its member oriented axes X_m and Y_m. Since the joint stiffness matrix S_j is based upon axes oriented to the structure, it also becomes necessary to obtain member stiffnesses for the structure. In the computer program, the member stiffness matrix for a member is generated, and then it is transformed to the structure oriented axes by a process of rotation of axes. Using an appropriate transformation matrix, the rotation of axes may be executed by matrix multiplications. The subject of the organization and application of the rotation and transformation matrices is far too lengthy for the content of this report. For a full explanation of transformation and rotation matrices, the references (Analysis of Framed Structures) in the bibliography should prove adequate. Direction cosines of the member axis with respect to the structure oriented axis are used in setting up the rotation matrices.

As an initial step in the analysis of a plane truss, all of the joints and members must be numbered. After the numbering is complete, it is necessary to record the two joint numbers that are associated with each member. This association of joint numbers with member numbers is necessary in order to ascertain which elements of the joint stiffness matrix S_j and which load vectors receive contributions from each member. It also is necessary to identify a j and k end of each member, so that the origin of the member oriented axes X_m and Y_m may be set at the j end. All possible joint displacements and degrees of freedom must be identified, to set up the joint stiffness matrix, as previously mentioned.

Load vectors must be arranged in matrix form also. First, the loads that are applied directly to the joints are read into the computer, and then the member end actions from loads applied on the members are read in. These two loading conditions are then combined to form a single loading condition applied at the joints of the truss.

The final matrix arrangement to be solved by the computer is shown below.

$$R_T \; A = S_m \; R_T \; D$$

$$A = R_T^{-1} \; S_m \; R_T \; D$$

R_T = transformation rotation matrix

S_m = joint stiffness matrix, member oriented

R_T^{1} = inverse of R

A = joint loading conditions, oriented to structure axes

D = final unknown joint displacements

After the unknown joint displacements have been solved for, the the actual member end actions, and the support reactions, are solved for by plugging the displacements into the over-all joint stiffness matrix. This only represents a very brief outline of a process about which many books are devoted to the explanation. This should only serve as a general guide for the reader to understand what the program is doing. For a better understanding of the manipulation of these matrices and processes by the computer, refer to the printed copy of the source program.

A general flow chart of the program is presented next.

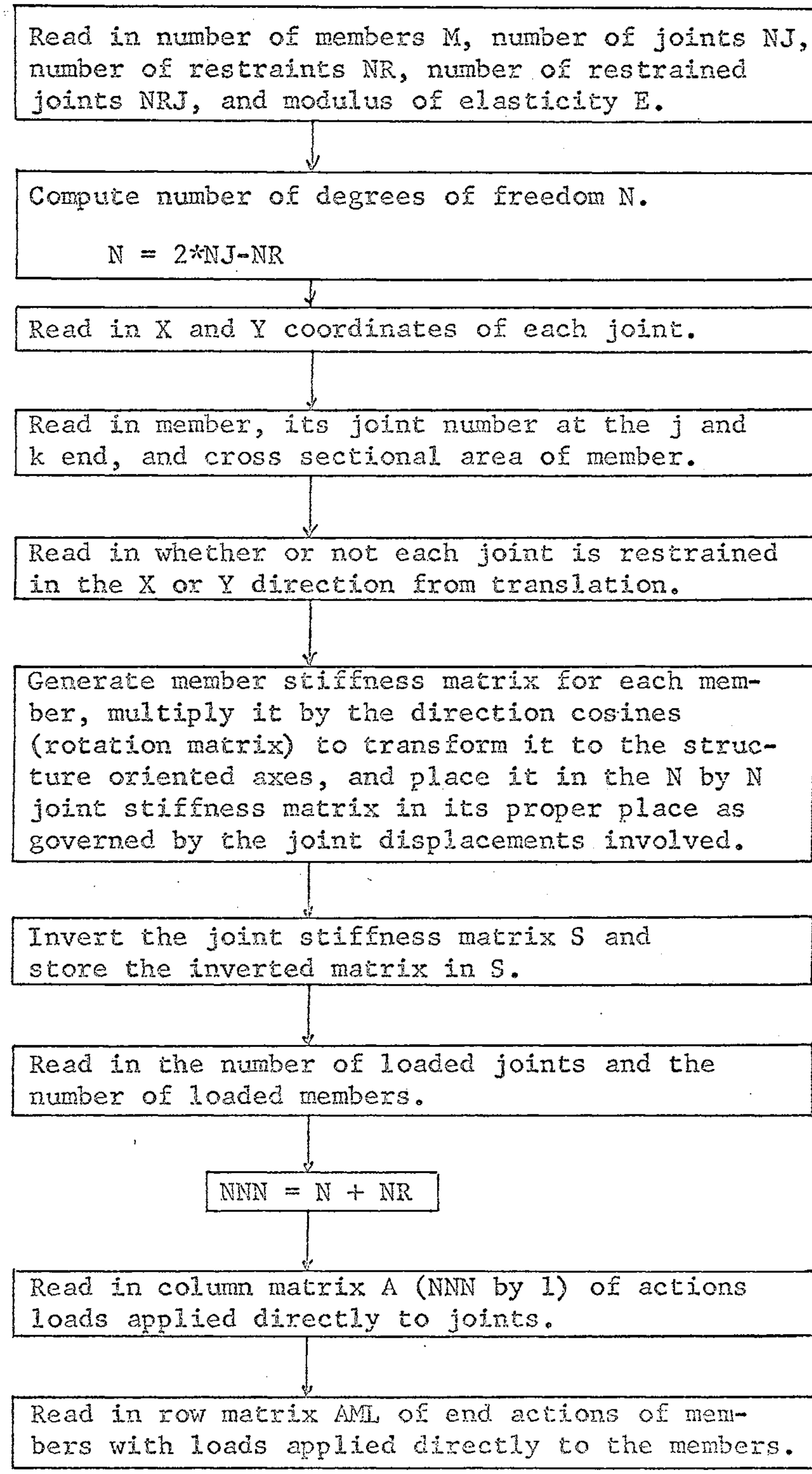


Figure 20.

Flow Chart for Plane Truss by the Stiffness Method

Sum member and actions AML applied to joints, for each individual joint.

Combine joint loads to find final load AC applied to each joint.

```
AC - A + AE
```

Find joint displacements D(J) for joints J.

```
        DO 111 J = 1,N
        DO 110 K = 1,N
        D(J) = D(J) + S(J,K)*AC(K)
110     CONTINUE
111     CONTINUE
```

Find support reactions AR(K) for joints K.

```
        N1 = N + 1
        DO 113 K = N1,NNN
        AR(K) = -AC(K)
        DO 112 J = 1,N
        AR(K) = AR(K) + S(K,J)*D(J)
112     CONTINUE
113     CONTINUE
```

Punch out joint displacements and reactions.

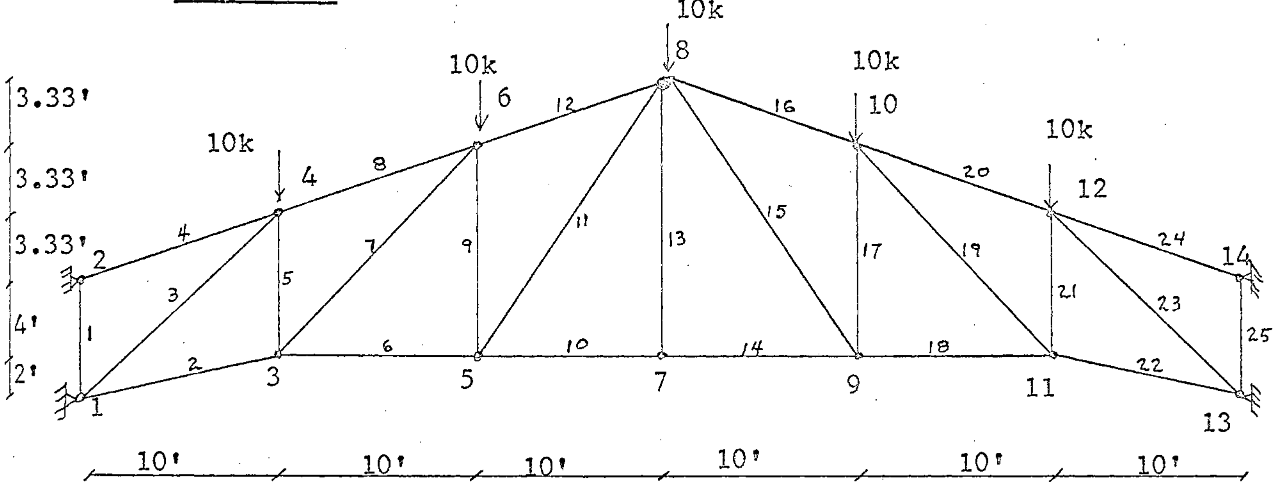
Compute the final member end actions from the sum of the applied member end actions from the applied loads plus the individual member stiffness matrix times the structure oriented joint displacements times the direction cosines.

Punch out the final member end actions.

Figure 21.

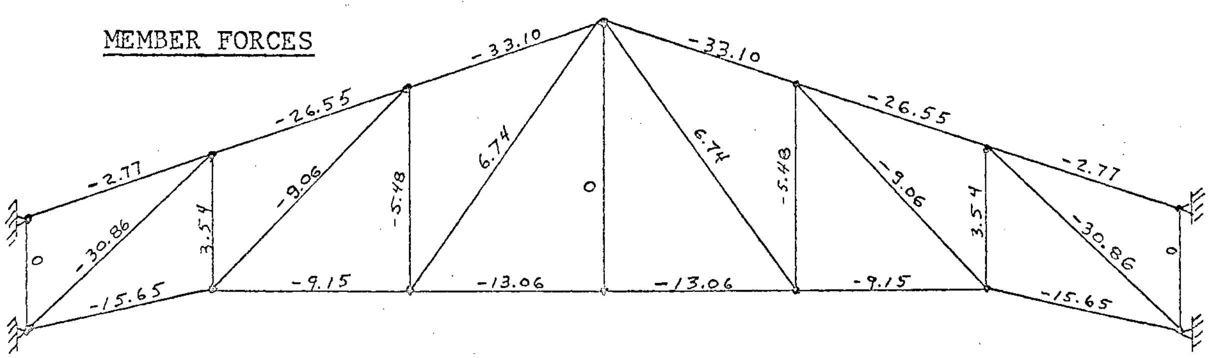
Truss Flow Chart Continued

TRUSS ARCH



Note joint and member numbers

MEMBER FORCES



Note that most members of the truss are in compression due to the arching action.

Figure 22.

Sample Problem - Truss Arch

TABLE IX
INPUT DATA FOR TRUSS

25	14	8	4	30000.0
0.0		0.0		
0.0		6.00		
10.0		2.0		
10.0		9.33		
20.0		2.0		
20.0		12.66		
30.0		2.0		
30.0		16.00		
40.0		2.00		
40.0		12.66		
50.0		2.00		
50.0		9.33		
60.0		0.0		
60.0		6.0		
1	2	2.00		
1	3	4.00		
1	4	2.00		
2	4	4.00		
3	4	2.0		
3	5	4.00		
3	6	2.00		
4	6	4.00		
5	6	2.00		
5	7	4.00		
5	8	2.00		
6	8	4.00		
7	8	2.00		
7	9	4.00		
8	9	2.00		
8	10	4.00		
9	10	2.00		
9	11	4.00		
10	11	2.00		
10	12	4.00		
11	12	2.00		
11	13	4.00		
12	13	2.00		
12	14	4.00		
13	14	2.00		
1				
1	1			
2				
1	1			

13		
1	1	
14		
1	1	
5	0	
4		
0.0		-10.0
6		
0.0		-10.0
8		
0.0		-10.0
10		
0.0		-10.0
12		
0.0		-10.0

TABLE X
FINAL OUTPUT FOR TRUSS

$JOB BEV EDWARDS 2509-40040

ANALYSIS OF PLANE TRUSSES
STRUCTURE DATA

M	N	NJ	NR	NRJ	E
25	20	14	8	4	30000.000

COORDINATES OF JOINTS

JOINT	X	Y
1	0.000	0.000
2	00.000	6.000
3	10.000	2.000
4	10.000	9.330
5	20.000	2.000
6	20.000	12.660
7	30.000	2.000
8	30.000	16.000
9	40.000	2.000
10	40.000	12.660
11	50.000	2.000
12	50.000	9.330
13	60.000	0.000
14	60.000	6.000

MEMBER DESIGNATIONS, AREAS, LENGTHS, AND DIRECTION COSINES

MEMBER	JJ	JK	AREA	LENGTH	CX	CY
1	1	2	2.000	6.000	0.000	1.000
2	1	3	4.000	10.198	0;981	0.196
3	1	4	2.000	13.677	0.731	0.682
4	2	4	4.000	10.540	0.949	0.316
5	3	4	2.000	7.330	0.000	1.000
6	3	5	4.000	10.000	1.000	0.000
7	3	6	2.000	14.616	0.684	0.729
8	4	6	4.000	10.540	0.949	0.316
9	5	6	2.000	10.660	0.000	1.000
10	5	7	4.000	10.000	1.000	0.000
11	5	8	2.000	17.205	0.581	0.814
12	6	8	4.000	10.543	0.948	0.317
13	7	8	2.000	14.000	0.000	1.000
14	7	9	4.000	10.000	1.000	0.000
15	8	9	2.000	17.205	0.581	-0.814
16	8	10	4.000	10.543	0.948	-0.317
17	9	10	2.000	10.660	0.000	1.000

18	9	11	4.000	10.000	1.000	0.000
19	10	11	2.000	14.616	0.684	-0.729
20	10	12	4.000	10.540	0.949	-0.315
21	11	12	2.000	7.330	0.000	1.000
22	11	13	4.000	10.540	0.981	-0.196
23	12	13	2.000	13.677	0.731	-0.682
24	12	14	4.000	10.540	0.949	-0.316
25	13	14	2.000	6.000	0.000	1.000

JOINT RESTRAINTS

JOINT	X RSTRT.	Y RSTRT.
1	1	1
2	1	1
13	1	1
14	1	1

LOAD DATA

NLJ	NLM
5	0

ACTIONS APPLIED AT JOINTS

JOINT	X ACTION	Y ACTION
4	0.000	-10.000
6	0.000	-10.000
8	0.000	-10.000
10	0.000	-10.000
12	0.000	-10.000

JOINT DISPLACEMENTS AND SUPPORT REACTIONS

JOINT	X DISPL.	Y DISPL.	X REAC.	Y REAC.
1	0.000	0.000	37.917	24.125
2	0.000	0.000	2.629	0.875
3	0.002	-0.016	0.000	0.000
4	0.005	-0.016	0.000	0.000
5	0.001	-0.021	0.000	0.000
6	0.004	-0.022	0.000	0.000
7	0.000	-0.017	0.000	0.000
8	-0.000	-0.017	0.000	0.000
9	-0.001	-0.021	0.000	0.000
10	-0.004	-0.022	0.000	0.000
11	-0.002	-0.016	0.000	0.000
12	-0.005	-0.016	0.000	0.000
13	0.000	0.000	-37.917	24.125
14	0.000	0.000	-2.629	0.875

MEMBER END ACTIONS

MEMBER	AM1	AM2	AM3	AM4
1	0.000	0.000	-0.000	0.000
2	15.654	0.000	-15.654	0.000
3	30.863	0.000	-30.863	0.000
4	2.771	0.000	-2.771	0.000
5	-3.540	0.000	3.540	0.000
6	9.150	0.000	-0.150	0.000
7	9.063	0.000	-0.063	0.000
8	26.556	0.000	-26.556	0.000
9	5.486	0.000	-5.486	0.000
10	13.068	0.000	-13.068	0.000
11	-6.742	0.000	6.742	0.000
12	33.101	0.000	-33.101	0.000
13	0.000	0.000	-0.000	0.000
14	13.068	0.000	-13.068	0.000
15	-6.742	0.000	6.742	0.000
16	33.101	0.000	-33.101	0.000
17	5.486	0.000	-5.486	0.000
18	9.150	0.000	-9.150	0.000
19	9.063	0.000	-9.063	0.000
20	26.556	0.000	-26.556	0.000
21	-3.540	0.000	3.540	0.000
22	15.654	0.000	-15.654	0.000
23	30.863	0.000	-30.863	0.000
24	2.771	0.000	-2.771	0.000
25	0.000	0.000	-0.000	0.000

Plane Frame Analysis by the Stiffness Method

The plane frame computer program, written by the author, is capable of determining the end moments, shears, and thrusts in each member of a two dimensional plane rigid frame of any member arrangement. The term "any member arrangement" means that the program is not restricted to frames of orthogonal member arrangement, but it may handle members which are not parallel to the X and Y structure oriented axes. As in the case of the plane truss, this requires the use of member oriented axes and rotation matrices to transfer the member stiffness matrices from the member oriented axes to the structure oriented axes. The member end actions due to member loads must likewise be transferred from the member to the structure oriented axes.

The basic operation of this stiffness method program is almost identical to that of the other stiffness method programs such as the plane truss program just presented. A general flow chart for the plane frame program would be the same as that of the plane truss.

The plane frame program differs from the plane truss in its member stiffness matrix, its rotation matrix, and its possible displacements at each joint. A plane frame has three possible unknown displacements at each joint. They are translation in the X direction, translation in the Y direction, and rotation about the Z axis. Each member of the structure may have up to six unknown displacements, three at each end. At the structure supports some of these displacements will be restrained, of course. A typical member, with its six possible displacements is shown, and its relation to the structure oriented axes should be noted. Single headed arrows represent translation, and double headed arrows represent rotation.

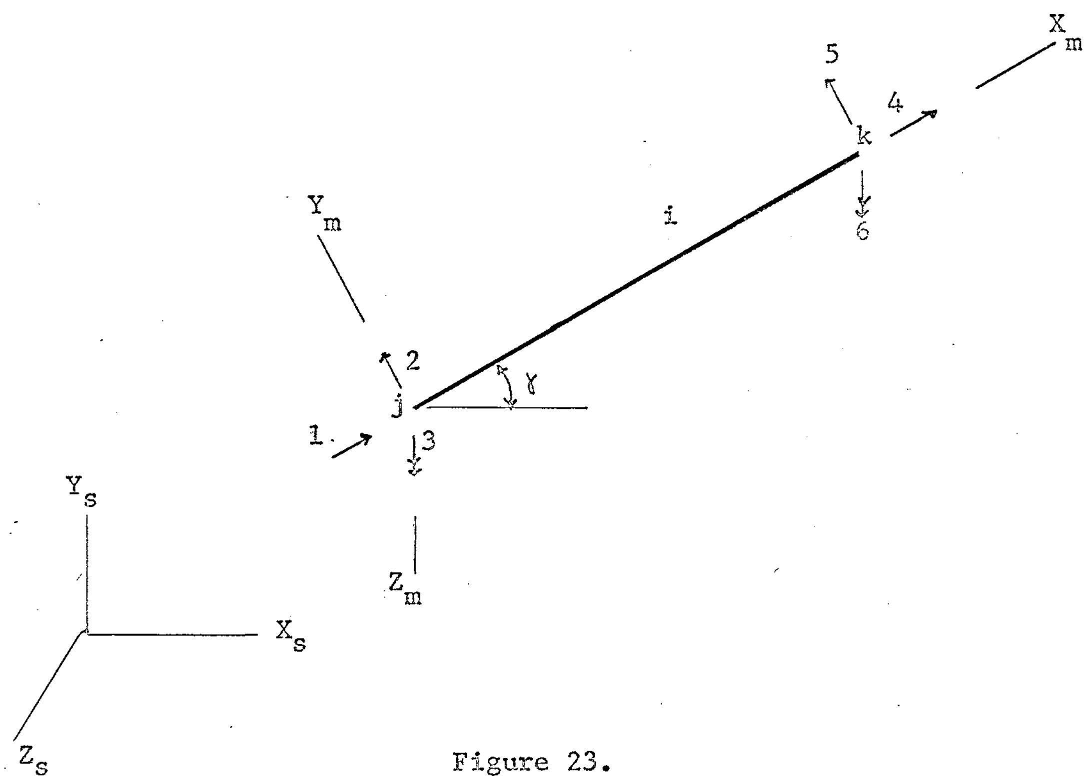


Figure 23.

Numbering System for a plane frame member.[1]

As in the case of the plane truss, each member and each joint must be numbered, and a j and k end must be assigned to each member so that the member oriented axes may be set up.

Since there are six possible degrees of freedom for each member, a 6 by 6 member stiffness matrix results. It is assumed that the reader has

[1] J.M. Gere and W. Weaver, Jr., Analysis of Framed Structures, (D. Van Nostrand Company, Inc.,) p. 259, Figure 4-27.

a knowledge of the derivation of the various stiffness factors present in the member stiffness matrix. The plane frame member stiffness matrix for member axes is shown below.

$$S_m = \begin{bmatrix} EA_x/L & 0 & 0 & -EA_x/L & 0 & 0 \\ 0 & 12EI_z/L^3 & 6EI_z/L^2 & 0 & -12EI_z/L^3 & 6EI_z/L^2 \\ 0 & 6EI_z/L^2 & 4\,EI_z/L & 0 & -6EI_z/L^2 & 2EI_z/L \\ -EA_x/L & 0 & 0 & EA_x/L & 0 & 0 \\ 0 & -12EI_z/L^3 & -6EI_z/L^2 & 0 & 12EI_z/L^3 & -6EI_z/L^2 \\ 0 & 6EI_z/L^2 & 2EI_z/L & 0 & -6EI_z/L^2 & 4EI_z/L \end{bmatrix}$$

Plane Frame Member Stiffness Matrix for Member Axes.

The member stiffness matrix must be transformed to the stiffness matrix for the structure axes. This requires the use of the rotation transformation matrix, R_T. As the first step in forming the rotation transformation matrix, the rotation matrix R is expressed in terms of the direction cosines. The 3 by 3 rotation matrix R is shown below.

$$R = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From the rotation matrix R the rotation transformation matrix R_T is made up.

$$R_T = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}$$

Once the transformation rotation matrix R_T is available, then the member stiffness matrix for the structure oriented axes may be found.

$$S_{MD} = R'_T \cdot S_m \cdot R_T$$

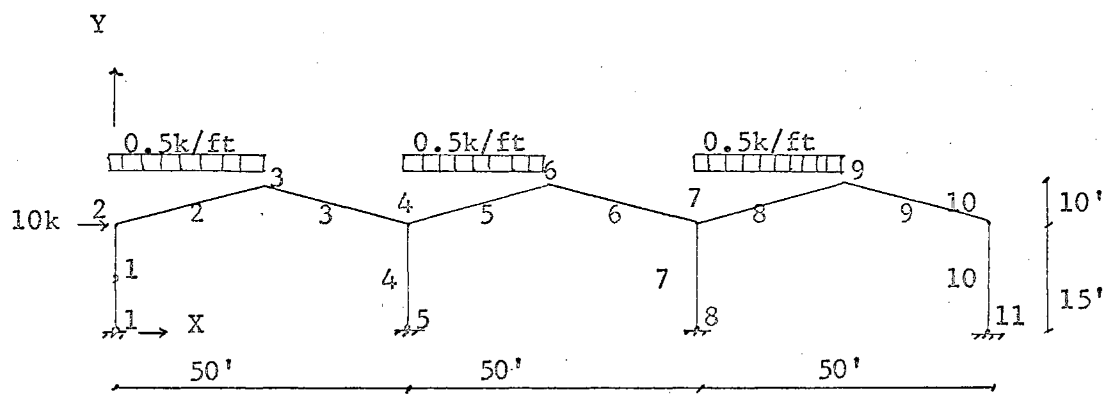
Due to lack of space, the 6 by 6 member stiffness matrix for the structure oriented axes, S_{MD} is not illustrated in full, but it should be clearly defined by the matrix multiplication shown. The member stiffness matric for each member is made up in this manner and transferred to the over-all joint stiffness matrix S_J , as was done with the plane truss.

The over-all joint stiffness matrix will be 3*NJ by 3*NJ, where NJ is the number of joints in the structure. The number of degrees of freedom N, is N = 3*NJ-NR, where NR is the number of joint restraint conditions. The joint stiffness matrix S to be inverted is of the order N by N. The over-all joint stiffness matrix is then used to find the member end actions and support reactions.

The input data for this program is very easy to prepare -- far easier than the virtual work program, by comparison. The input data consists of the number of members, number of joints, number of restrained joints, number of restraints, modulus of elasticity of the material, the X, Y coordinates of the joints, the properties of the members, the joint number on each end of a member, the joint restraints, and the loads applied to the structure. The input data for the sample problem took about 45 minutes to prepare and punch.

The program is currently set up for the IBM 7040 computer, but it can be broken down into several passes to run on smaller computers. It has been run on the IBm 1620 computer, but the lack of adequate storage on this computer cripples the program for anything but small structures, with few degrees of freedom. Versions of this program have recently appeared in the IBM 1130 User's Group, and it is very popular with practicing engineers. The sample problem should fully illustrate its merits.

THREE BAY GABLE FRAME WITH WIND AND DRIFT LOAD



BENDING MOMENTS (in-kips)

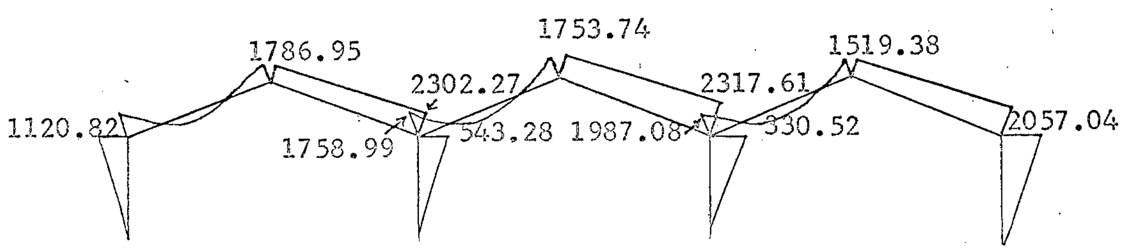


Figure 24.

Sample Problem - Three Bay Gable Frame

TABLE XI
INPUT DATA FOR GABLE FRAME

10		8 4	29000.0			
0.0		0.0				
0.0		180.0				
300.0		300.0				
600.0		180.0				
600.0		0.0				
900.0		300.0				
1200.0		180.0				
1200.0		0.0				
1500.0		300.0				
1800.0		180.0				
1800.0		0.0				
1	2	20.0	2000.0			
2	3	20.0	2000.0			
3	4	20.0	2000.0			
5	4	20.0	2000.0			
4	6	20.0	2000.0			
6	7	20.0	2000.0			
8	7	20.0	2000.0			
7	9	20.0	2000.0			
9	10	20.0	2000.0			
11	10	20.0	2000.0			
1						
1	1	0				
5						
1	1	0				
8						
1	1	0				
11						
1	1	0				
1	3					
2						
10.0		0.0	0.0			
2						
4.63		11.60	3750.0	4.63	11.60	-3750.0
5						
4.63		11.60	3750.0	4.63	11.60	-3750.0
8						
4.63		11.60	3750.0	4.63	11.60	-3750.0

TABLE XII
OUTPUT FOR GABLE FRAME

$JOB BEV EDWARDS 2509-40040

ANALYSIS OF PLANE FRAMES BY THE STIFFNESS METHOD

STRUCTURE DATA

M	N	NJ	NR	NRJ	E
10	25	11	8	3	29000.000

COORDINATES OF JOINTS

JOINT	X	Y
1	0.000	0.000
2	0.000	180.000
3	300.000	300.000
4	600.000	180.000
5	600.000	0.000
6	900.000	300.000
7	1200.000	180.000
8	1200.000	0.000
9	1500.000	300.000
10	1800.000	180.000
11	1800.000	0.000

MEMBER DESIGNATIONS AND PROPERTIES

	J	JK	AREA	MOM INERTIA	LENGTH	CX	CY
1	1	2	20.000	2000.000	180.000	0.000	1.000
2	2	3	20.000	2000.000	323.110	0.928	0.371
3	3	4	20.000	2000.000	323.110	0.928	-0.371
4	5	4	20.000	2000.000	180.000	0.000	1.000
5	4	6	20.000	2000.000	323.110	0.928	0.371
6	6	7	20.000	2000.000	323.110	0.928	-0.371
7	8	7	20.000	2000.000	180.000	0.000	1.000
8	7	9	20.000	2000.000	323.110	0.928	0.371
9	9	10	20.000	2000.000	323.110	0.928	-0.371
10	11	10	20.000	2000.000	180.000	0.000	1.000

JOINT RESTRAINTS

JOINT	X RSTRT.	Y RSTRT.	Z RSTRT.
1	1	1	0
5	1	1	0
8	1	1	0
11	1	1	0

LOAD DATA

NLJ	NLM
1	3

ACTIONS APPLIED AT JOINTS

JOINT	X ACTION	Y ACTION	Z ACTION
2	10.000	0.000	0.000

ACTIONS AT ENDS OF RESTRAINED MEMBERS DUE TO LOADS

MEMBER	AML1	AML2	AML3	AML4	AML5	AML6
2	4.63	11.60	3750.00	4.63	11.60	-3750.00

MEMBER	AML1	AML2	AML3	AML4	AML5	AML6
5	4.63	11.60	3750.00	4.63	11.60	-3750.00

MEMBER	AML1	AML2	AML3	AML4	AML5	AML6
8	4.63	11.60	3750.00	4.63	11.60	-3750.00

JOINT DISPLACEMENTS AND SUPPORT REACTIONS

JOINT	X DISPL	Y DISPL	Z DISPL	X REAC	Y REAC	Z REAC
1	0.00	0.00	-0.01	6.23	16.76	0.00
2	1.06	-0.01	-0.01	0.00	0.00	0.00
3	1.09	-0.10	0.01	0.00	0.00	0.00
4	1.11	-0.01	-0.01	0.00	0.00	0.00
5	0.00	0.00	-0.01	-3.02	26.02	0.00
6	1.12	-0.03	0.01	0.00	0.00	0.00
7	1.12	-0.01	-0.01	0.00	0.00	0.00
8	0.00	0.00	-0.01	-1.84	25.79	0.00
9	1.17	-0.16	0.01	0.00	6.23	16.76
10	1.23	-0.0	-0.00	-3.02	26.02	-1.84
11	0.00	0.0	-0.01	-11.43	6.36	6.36

MEMBER END ACTIONS

	END ACTIONS ON J END			END ACTIONS ON K END		
MEMBER	THRUST	SHEAR	MOMENT	THRUST	SHEAR	MOMENT
1	16.76	-6.23	-0.00	-16.76	6.23	-1120.82
2	21.29	9.54	1120.82	-12.03	13.66	-1786.95
3	18.13	-1.59	1786.95	-18.13	1.59	-2302.27
4	26.02	3.02	0.00	-26.02	-3.02	543.28
5	18.89	11.62	1758.99	-9.63	11.58	-1753.74
6	14.96	-1.75	1753.74	-14.96	1.75	-2317.61
7	25.79	1.84	-0.00	-25.79	-1.84	330.52
8	17.51	13.05	1987.08	-8.25	10.15	-1519.38
9	11.97	-1.66	1519.38	-12.97	1.66	-2057.04
10	6.36	11.43	-0.00	-6.36	-11.43	2057.04

Space Truss Analysis by the Stiffness Method

The space truss, or three dimensional truss, has members which are skewed with respect to the X,Y,Z axes of the structure. With pinned joints at both ends of a member, there are six possible unknown displacements for each member. Each joint may translate in the X,Y, or Z direction, with no rotation of restraint considered to be present. A space truss with J joints will have 3*J possible degrees of freedom, less the support restraints.

The basic flow charts for the plane truss, the plane frame, and the space frame programs are presented in <u>Analysis of Framed Structures</u>, by J.M. Gere and W. Weaver, Jr. The original author of the space truss program is unknown. All that was available to the author of this report was a working version of the program, with no commentary available. Gere and Weaver present flow charts for a space truss program, which is quite similar to the program presented here. This program has a slight advantage over that of Gere and Weaver, in that it contains a routine for maximizing the forces present in the members for a combination of different loading conditions. In other words, if several different types of loading conditions are applied separately to the same structure, the program will select the loading combination for each member that will cause the maximum tension or comptession in the member. This maximum value is punched out. This is an especially handy feature for a designer to use in determining the worst loading conditions for a structure.

The stiffness method approach to a space frame problem is basically the same as that for the plane frame. The member stiffness matrix is of the order 6 by 6, since there are six possible unknown displacements for each member. The rotation matrix is slightly different also. The space truss member stiffness matrix for the member oriented axes is shown below.[1]

$$S_m = EA/L \cdot \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The direction cosines, for use in the rotation matrix, are generated from the coordinates of the j and k ends of the member, in the structure oriented axes system.

$$C_x = (x_k - x_j)/L$$
$$C_y = (y_k - y_j)/L$$
$$C_z = (z_k - z_j)/L$$

$$L = \sqrt{(x_k - x_j)^2 + (y_k - y_j)^2 + (z_k - z_j)^2}$$

After the rotation matrices are generated and combined to form the rotation transformation matrix, they are applied to the member stiffness matrix in the member axes system to form the member stiffness matrix in the structure oriented axes system, for the space truss. The basic space truss member stiffness matrix for the structure axes system is shown on the following page.

[1] J. M. Gere and W. Weaver, Jr., Analysis of Framed Structures, p. 280, Figure 4-39.

$$S_{MD} = EA_x/L \cdot \begin{bmatrix} C_x^2 & C_yC_x & C_zC_x & -C_x^2 & -C_yC_x & -C_zC_x \\ C_xC_y & C_y^2 & C_zC_y & -C_xC_y & -C_y^2 & -C_zC_y \\ C_xC_z & C_yC_z & C_z^2 & -C_xC_z & -C_yC_z & -C_z^2 \\ -C_x^2 & -C_yC_x & -C_zC_x & C_x^2 & C_yC_x & C_zC_x \\ -C_xC_y & -C_y^2 & -C_zC_y & C_xC_y & C_y^2 & C_zC_y \\ -C_xC_z & -C_yC_z & C_z^2 & C_xC_z & C_yC_z & C_z^2 \end{bmatrix}$$

As in the other stiffness method programs, the member stiffness matrices are combined to form the over-all joint stiffness matric, which is inverted. Loading conditions are read in and the unknown joint displacements are solved for. The final member forces are solved for, and the results are punched out.

A sample problem was run for a Schwedler dome, with wind and gravity load applied separately. The input data for the program is simple to prepare. The number of members, number of joints, number of joints restrained, and number of loading conditions are read in first. Next, the joint coordinates and numbers of the joints on each end of a member are read in. After the stiffness matrix has been inverted, the loads applied to each joint are read in. This input data is all clearly explained in the comment statements in the source program. The program is well suited for the analysis of domes, transmission towers, and other such space trusses.

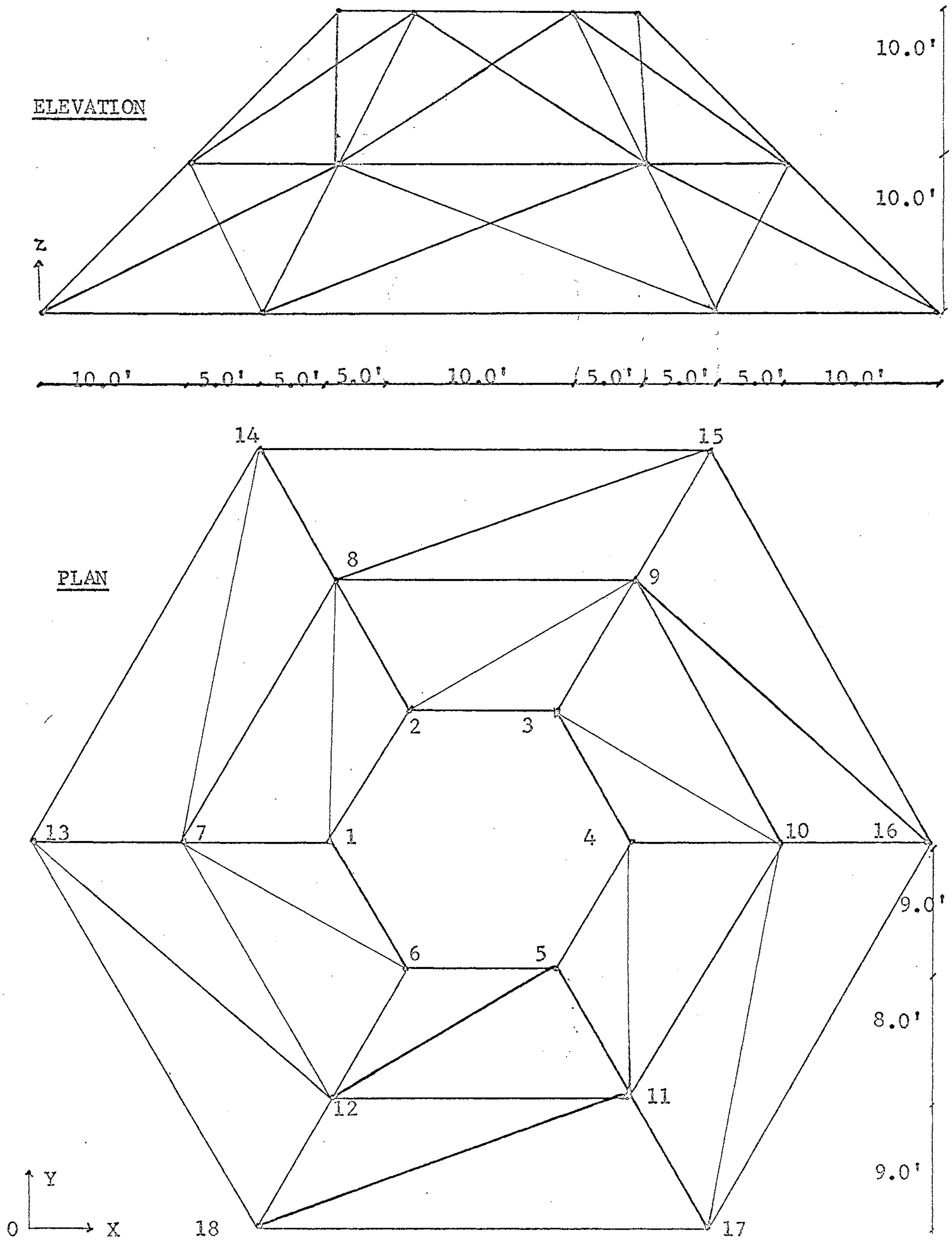
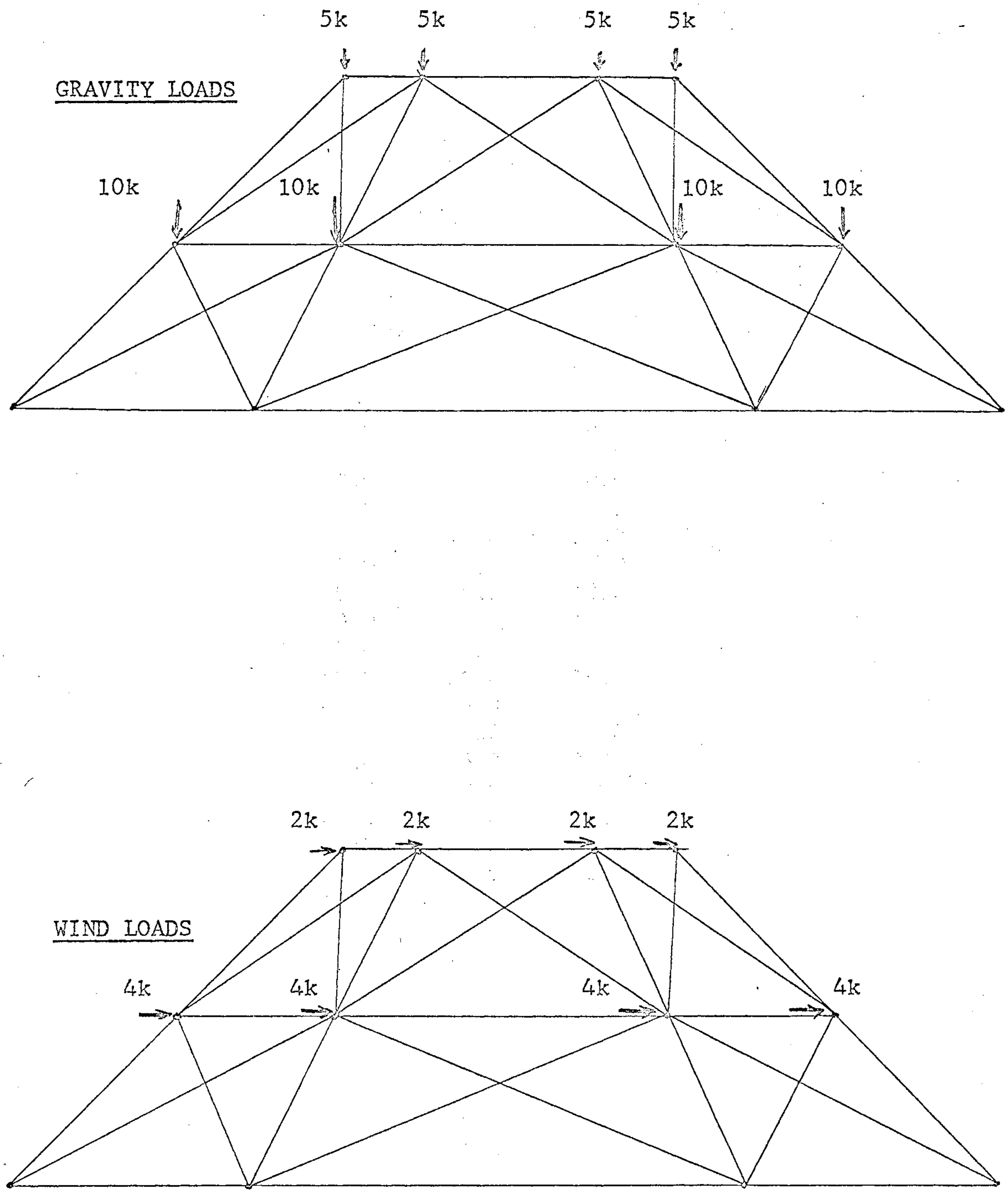


Figure 25.

Sample Problem - Schwedler Dome



Gravity and wind loads are applied to joints 1 - 12 of the structure.

Figure 26.

Loading Conditions for Schwedler Dome

TABLE XIII

SAMPLE PROBLEM INPUT DATA FOR SCHWEDLER DOME

SCHWEDLER DOME ANALYSIS FOR GRAVITY AND WIND LOAD

18064202		
20.0	26.0	20.0
25.0	35.0	20.0
35.0	35.0	20.0
40.0	26.0	20.0
35.0	17.0	20.0
25.0	17.0	20.0
10.0	26.0	10.0
20.0	43.0	10.0
40.0	43.0	10.0
50.0	26.0	10.0
40.0	9.0	10.0
20.0	9.0	10.0
0.0	26.0	0.0
15.0	52.0	0.0
45.0	52.0	0.0
60.0	26.0	0.0
45.0	0.0	0.0
15.0	0.0	0.0
1.0	0102	
1.0	0203	
1.0	0304	
1.0	0405	
1.0	0506	
1.0	0601	
1.0	0708	
1.0	0809	
1.0	0910	
1.0	1011	
1.0	1112	
1.0	1207	
1.0	1314	
1.0	1415	
1.0	1516	
1.0	1617	
1.0	1718	
1.0	1813	
1.0	0208	
1.0	0309	

1.0	0410	
1.0	0511	
1.0	0612	
1.0	0107	
1.0	0713	
1.0	0814	
1.0	0915	
1.0	1016	
1.0	1117	
1.0	1218	
1.0	0108	
1.0	0209	
1.0	0310	
1.0	0411	
1.0	0512	
1.0	0607	
1.0	0714	
1.0	0815	
1.0	0916	
1.0	1017	
1.0	1118	
1.0	1213	
0.0	0.0	-5.0
0.0	0.0	-5.0
0.0	0.0	-5.0
0.0	0.0	-5.0
0.0	0.0	-5.0
0.0	0.0	-5.0
0.0	0.0	-10.0
0.0	0.0	-10.0
0.0	0.0	-10.0
0.0	0.0	-10.0
0.0	0.0	-10.0
0.0	0.0	-10.0
2.0	0.0	0.0
2.0	0.0	0.0
2.0	0.0	0.0
2.0	0.0	0.0
2.0	0.0	0.0
2.0	0.0	0.0
4.0	0.0	0.0
4.0	0.0	0.0
4.0	0.0	0.0
4.0	0.0	0.0
4.0	0.0	0.0
4.0	0.0	0.0

TABLE XIV

OUTPUT FOR SCHWEDLER DOME

$JOB
1 BEV EDWARDS 2509-40040

SCHWEDLER DOME ANALYSIS FOR GRAVITY AND WIND LOAD

ANALYSIS FOR LOADING CONDITION NUMBER 1

DISPLACEMENTS

JOINT	X-DISPL	Y-DISPL	Z-DISPL
1	0.039	-0.123	-0.259
2	-0.123	-0.259	-0.105
3	-0.259	-0.105	-0.064
4	-0.105	-0.064	-0.245
5	-0.064	-0.245	-0.124
6	-0.245	-0.124	0.052
7	-0.124	0.052	-0.255
8	0.052	-0.255	-0.039
9	-0.255	-0.039	0.123
10	-0.039	0.123	-0.259
11	0.123	-0.259	0.105
12	-0.259	0.105	0.064

MEMBER	STRESS
1 - 2	-4.58
2 - 3	-4.71
3 - 4	-5.13
4 - 5	-4.58
5 - 6	-4.71
6 - 1	-5.13
7 - 8	-10.76
8 - 9	-10.59
9 - 10	-10.14
10 - 11	-10.76
11 - 12	-10.59
12 - 7	-10.14
13 - 14	-0.00
14 - 15	-0.00
15 - 16	-0.00
16 - 17	0.00
17 - 18	0.00
18 - 13	0.00

2	- 8	-6.86
3	- 9	-7.27
4	- 10	-6.67
5	- 11	-6.86
5	- 12	-7.27
1	- 7	-6.67
7	- 13	-20.83
8	- 14	-21.92
9	- 15	-21.52
10	- 16	-20.83
11	- 17	-21.92
12	- 18	-21.52
1	- 8	-0.56
2	- 9	-0.02
3	- 10	0.58
4	- 11	-0.56
5	- 12	-0.02
6	- 7	0.58
7	- 14	0.85
8	- 15	-0.01
9	- 16	-0.85
10	- 17	0.85
11	- 18	-0.01
12	- 13	-0.85

ANALYSIS FOR LOADING CONDITION NUMBER 2

DISPLACEMENTS

JOINT	X-DISPL	Y-DISPL	Z-DISPL
1	0.160	-0.029	-0.106
2	-0.029	-0.106	0.143
3	-0.106	0.143	-0.020
4	0.143	-0.020	-0.015
5	-0.020	-0.015	0.151
6	-0.015	0.151	-0.025
7	0.151	-0.025	0.078
8	-0.025	0.078	0.160
9	0.078	0.160	-0.029
10	0.160	-0.029	0.106
11	-0.029	0.106	0.143
12	0.106	0.143	-0.020

MEMBER		STRESS
1	- 2	0.00
2	- 3	1.94
3	- 4	2.00
4	- 5	0.00
5	- 6	-1.94
6	- 1	-2.00

7 - 8	1.18
8 - 9	5.97
9 - 10	5.04
10 - 11	-1.18
11 - 12	-5.97
12 - 7	-5.04
13 - 14	-0.00
14 - 15	-0.00
15 - 16	-0.00
16 - 17	0.00
17 - 18	0.00
18 - 13	0.00
2 - 8	2.71
3 - 9	1.41
4 - 10	-1.45
5 - 11	-2.71
6 - 12	-1.41
1 - 7	1.45
7 - 13	5.32
8 - 14	6.08
9 - 15	1.10
10 - 16	-5.32
11 - 17	-6.08
12 - 18	-1.10
1 - 8	-2.03
2 - 0	-3.89
3 - 10	-2.07
4 - 11	2.03
5 - 12	3.89
6 - 7	2.07
7 - 14	-4.83
8 - 15	-0.35
9 - 16	-4.79
10 - 17	4.83
11 - 18	9.35
12 - 13	4.79

MAXIMUM STRESSES UNDER ANY COMBINATION OF ABOVE

MEMBER	MAX. TENSION	MAX. COMPRESSION
1 - 2	0.00000	-4.57584
2 - 3	1.94286	-4.70635
3 - 4	2.00029	-5.13147
4 - 5	0.00000	-4.57583
5 - 6	0.00000	-6.64921
6 - 1	0.00000	-7.13176
7 - 8	1.18339	-10.75562
8 - 9	5.96616	-10.58867
9 - 10	5.03828	-10.14283

10 - 11	0.00000	-11.93900
11 - 12	0.00000	-16.55484
12 - 7	0.00000	-15.18110
13 - 14	0.00000	0.00000
14 - 15	0.00000	0.00000
15 - 16	0.00000	0.00000
16 - 17	0.00000	0.00000
17 - 18	0.00000	0.00000
18 - 13	0.00000	0.00000
2 - 8	2.71027	-6.86295
3 - 9	1.41405	-7.26665
4 - 10	0.00000	-8.12162
5 - 11	0.00000	-9.57322
6 - 12	0.00000	-8.68071
1 - 7	1.45462	-6.66700
7 - 13	5.32262	-20.83224
8 - 14	6.08029	-21.92415
9 - 15	1.09606	-21.51612
10 - 16	0.00000	-26.15486
11 - 17	0.00000	-28.00444
12 - 18	0.00000	-22.61218
1 - 8	0.00000	-2.59218
2 - 9	0.00000	-3.90392
3 - 10	0.57570	-2.07251
4 - 11	2-02866	-0.56352
5 - 12	3.88827	-0.01565
6 - 7	2.64821	0.00000
7 - 14	0.85486	-4.82978
8 - 15	0.00000	-9.35734
9 - 16	0.00000	-5.64361
10 - 17	5.68464	0.00000
11 - 18	9.35024	-0.00709
12 - 13	4.79346	-0.85015

Space Frame Analysis by the Stiffness Method

The space frame is usually the most complicated of all types of framed structures. A member in a space frame may have its member axes skewed with respect to the structure axes, as in the case of the space truss. The space frame is a rigidly framed three dimensional structure, which may have translation and rotation about the X, Y, Z axes. This means that each joint may have three translational and three rotational unknown displacements. Since a typical member i frames into a joint on each end (member has j and k end) it will have a total of twelve unknown displacements, six from each end. The sketch below illustrates the twelve unknown displacements.[1] A single headed arrow represents a translation, and a double headed arrow represents a rotation. These displacements are shown in the structure oriented axes system.

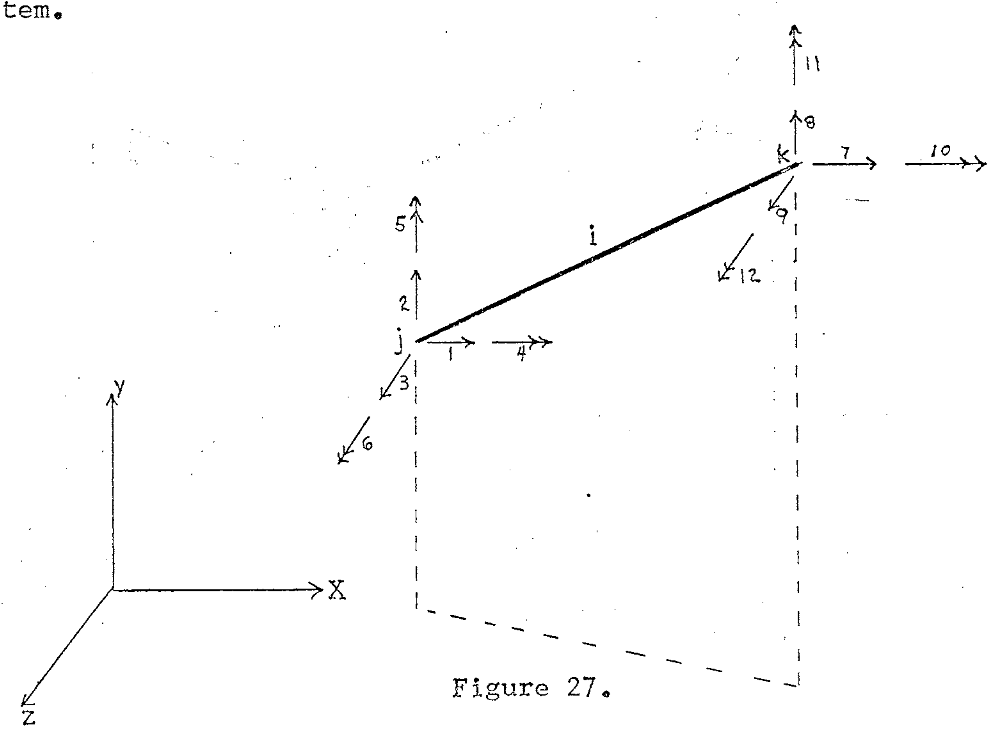


Figure 27.

Typical Space Frame Member

[1] J. M. Gere and W. Weaver, Jr. Analysis of Framed Structures, (D. Van Nostrand Company, Inc.,) p. 290, Figure 4-44.

The member also may be rotated about its own X_m axis, by an angle . A sketch of the rotation of a space frame member about its X_m axis is shown below.[2]

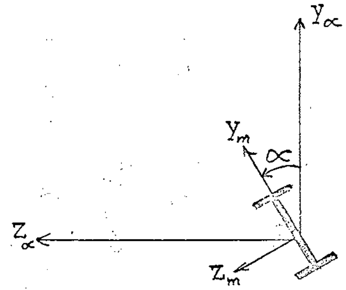


Figure 28.

Member Rotated by α About Its Own Axis

The final displacements of the member, in its own member oriented axes, after the member has been rotated the angle   (optional), are shown in Figure 29.[2]

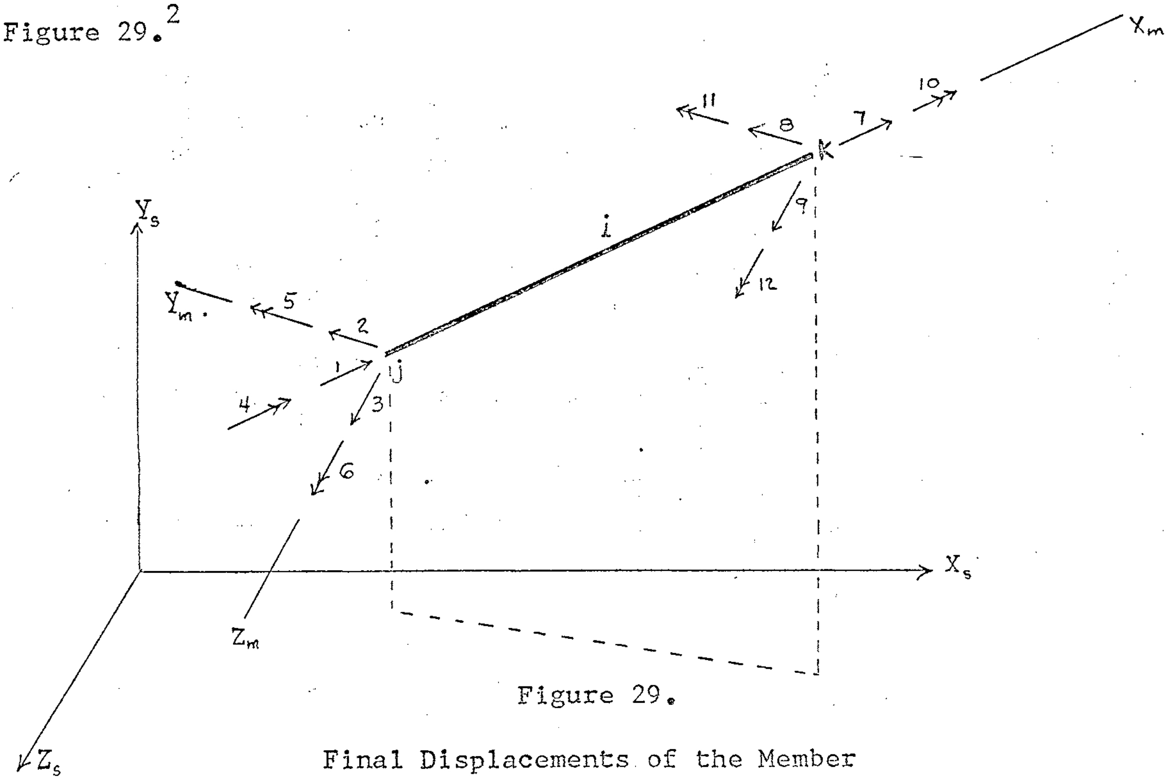


Figure 29.

Final Displacements of the Member

[2]J.M. Gere and W. Weaver, Jr., Analysis of Framed Structures, (D. Van Nostrand Company, Inc.,) p. 291, Figure 4-46

$$
S = \begin{bmatrix}
EA_x/L & 0 & 0 & 0 & 0 & 0 & -EA_x/L & 0 & 0 & 0 & 0 & 0 \\
0 & 12EI_z/L^3 & 0 & 0 & 0 & 6EI_z/L^2 & 0 & -12EI_z/L^3 & 0 & 0 & 0 & 6EI_z/L^2 \\
0 & 0 & 12EI_y/L^3 & 0 & -6EI_y/L^2 & 0 & 0 & 0 & -12EI_y/L^3 & 0 & -6EI_y/L^2 & 0 \\
0 & 0 & 0 & GI_x/L & 0 & 0 & 0 & 0 & 0 & -GI_x/L & 0 & 0 \\
0 & 0 & -6EI_y/L^2 & 0 & 4EI_y/L & 0 & 0 & 0 & 6EI_y/L^2 & 0 & 2EI_y/L & 0 \\
0 & 6EI_z/L^2 & 0 & 0 & 0 & 4EI_z/L & 0 & -6EI_z/L^2 & 0 & 0 & 0 & 2EI_z/L \\
-EA_x/L & 0 & 0 & 0 & 0 & 0 & EA_x/L & 0 & 0 & 0 & 0 & 0 \\
0 & -12EI_z/L^3 & 0 & 0 & 0 & -6EI_z/L^2 & 0 & 12EI_z/L^3 & 0 & 0 & 0 & -6EI_z/L^2 \\
0 & 0 & -12EI_y/L^3 & 0 & 6EI_y/L^2 & 0 & 0 & 0 & 12EI_y/L^3 & 0 & 6EI_y/L^2 & 0 \\
0 & 0 & 0 & -GI_x/L & 0 & 0 & 0 & 0 & 0 & GI_x/L & 0 & 0 \\
0 & 0 & -6EI_y/L^2 & 0 & 2EI_y/L & 0 & 0 & 0 & 6EI_y/L^2 & 0 & 4EI_y/L & 0 \\
0 & 6EI_z/L^2 & 0 & 0 & 0 & 2EI_z/L & 0 & -6EI_z/L^2 & 0 & 0 & 0 & 4EI_z/L
\end{bmatrix}
$$

Figure 30.

Space Frame Member Stiffness Matrix

The basic member stiffness matrix for the member oriented axes is shown in Figure 30. This is a 12 by 12 stiffness matrix, because of the 12 possible displacements per member. The transformation rotation matrix must be applied to the member stiffness matrix to transfer it to the structure oriented axes system, so that it can be placed in the over-all joint stiffness matrix.

The transformation rotation matrix is composed of several rotation matrices, as in the other stiffness method programs. The derivation of the rotation matrix is presented in several of the reference texts. The rotation matrix is illustrated below. The direction cosines C_x, C_y, C_z were derived in the previous discussion of the space truss rotation matrix. The rotation matrix for the space frame is as follows:

$$R = \begin{bmatrix} C_x & C_y & C_z \\ \dfrac{-(C_x C_y \cos\alpha - C_z \cdot \sin\alpha)}{\sqrt{C_x^2 + C_z^2}} & \sqrt{C_x^2 + C_z^2}\cos\alpha & \dfrac{-(C_y C_z \cos\alpha + C_x \sin\alpha)}{\sqrt{C_x^2 + C_z^2}} \\ \dfrac{(C_x C_y \sin\alpha - C_z \cdot \cos\alpha)}{\sqrt{-C_x^2 + C_z^2}} & \sqrt{C_x^2 + C_z^2}\sin\alpha & \dfrac{(C_y C_z \cdot \sin\alpha + C_x \cdot \cos\alpha)}{\sqrt{C_x^2 + C_z^2}} \end{bmatrix}$$

The transformation rotation matrix is generated from the rotation matrix as follows:

$$R_T = \begin{bmatrix} R & 0 & 0 & 0 \\ 0 & R & 0 & 0 \\ 0 & 0 & R & 0 \\ 0 & 0 & 0 & R \end{bmatrix}$$

The member stiffness matrix for the structure oriented axes is computed by the usual matrix multiplications:

$$S_{MD} = R'_T \cdot S_M \cdot T_T$$

The flow chart for the basic operation of the space frame program is the same as the other stiffness method programs. The over-all joint stiffness matrix is composed of the contributions from various member stiffness matrices, placed according to the degrees of freedom involved. The matrix is inverted, the loads are read in, the joint displacements are solved for, and the final member end actions are determined.

The space frame approach to the analysis of a rigidly framed structure is the most rigorous method available. No longer is it necessary to make the simplifying assumption of cutting a rigid frame into plane frame sections for the analysis. The total structure, in the case of a multi-story office building, for example, can be placed in the computer all at once, in three dimensions. This will give the best possible values of the member forces, for the assumed loading conditions. The space frame program includes the effect of axial force, shear, torsion, and triaxial bending moment in each member. Shear and torsion may not seem to be significant, but the effect of axial force may be interesting. An excellent example of the effect of axial force in the columns of a 20-story building has been investigated by some researchers.[3] A 20-story building was investigated with space frame analysis, by computer. A few of their conclusions are quoted here.

[3] William Weaver, Jr., and Mark Nelson, Three-Dimensional Analysis of Tier Buildings, (Vol. 92, NO. ST 6. A.S.C.E. Structural Division), Dec., 1966.

> "Tier buildings can be analyzed as space structures using the three-dimensional approach . . . This type of analysis becomes mandatory whenever the structure is not symmetrical. A digital computer is necessary to implement the solution.
>
> Analyses of a 20-story building indicate that neglecting axial strains in columns causes rigid body displacements to be too small (20% in the example.) Much more dramatic errors than this occur in the joint rotations and the member end actions in the upper part of the building, showing that the results are grossly erroneous when column strains are omitted. The omission of torsional rigidities in the same example results in displacements that are only slightly in error."

Thus, errors in excess of 20% were found for the member end moments in the problem studied by the researchers. This fact alone should demonstrate the merit of space frame analysis by computers.

Space frame analysis by computer is not without its difficulties, however. Since each unrestrained joint has six possible unknown displacements, the size of the joint stiffness matrix rapidly expands to the memory limit of most computers. Space frame analysis is not practical on anything but a large computer. To limit the number of possible displacements in a structure, torsion and shear may be ignored by coding this fact into the input data for the joint restraints. This will allow more joints to be considered (with less than six degrees of freedom), for the same maximum size of matrix allowed for a given computer. As the program is currently set up for the I.B.M. 7040, a maximum size matrix of about 175 by 175 could be handled. For a larger matrix, a more sophisticated method of handling the matrix is necessary. Since such a large matrix would exceed the allowable memory if placed in core all at once, parts must be temporarily stored on tape during the solution. Algorithms have been developed for this purpose, but they are beyond the scope of this report.

The I.B.M. 1130 Users Group and the 7090 Users Group offer a 'canned" space frame program called "STRESS", which has been used quite successfully by structural engineers for several years. The program "STRESS" was originally developed at Massachusetts Institute of Technology. Another commercial program, "FRAME", will size the wide-flange members for a space frame, after a preliminary analysis has been preformed with some trial member sizes.

A very simple sample problem has been run to illustrate the results of the program. The final output of the member end actions are in kips and inch-kips.

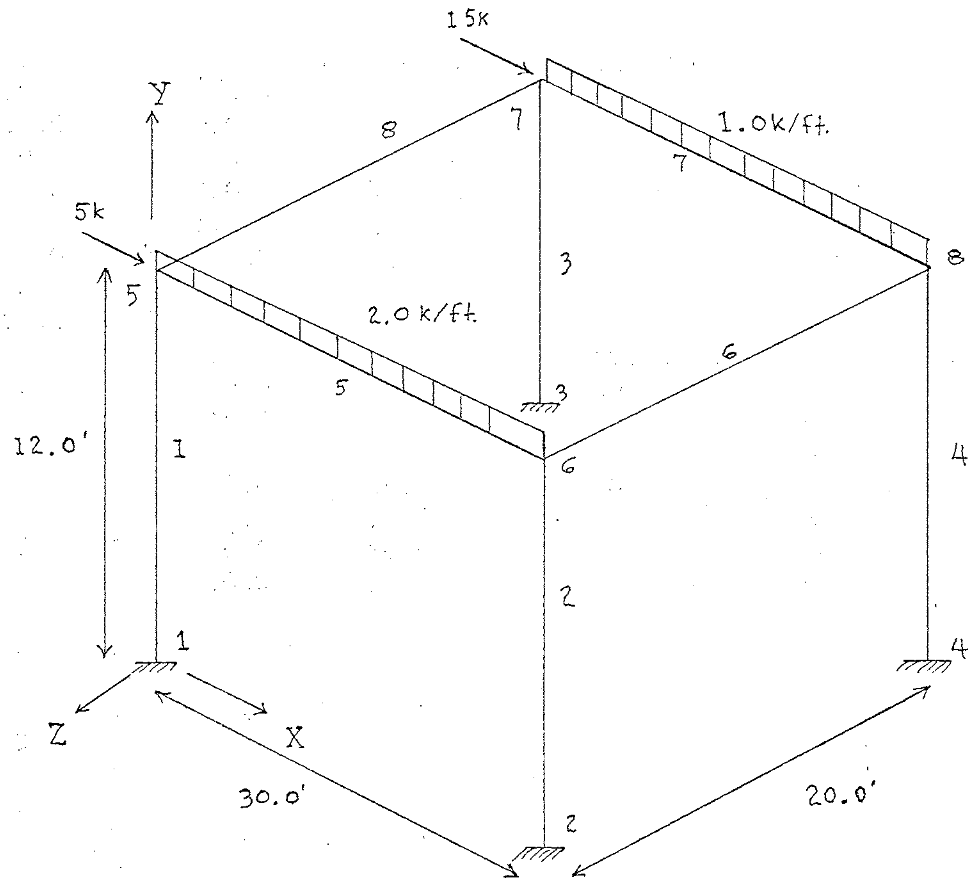


Figure 31.

Sample Space Frame Problem

TABLE XV

INPUT DATA FOR SPACE FRAME

8	8	24	4	29000.0		12000.0		
0.0		0.0		0.0				
360.0		0.0		0.0				
0.0		0.0		-240.0				
360.0		0.0		-240.0				
0.0		144.0		0.0				
360.0		144.0		0.0				
0.0		144.0		-240.0				
360.0		144.0		-240.0				
1	5	10.0		30.0		40.0	60.0	0.0
2	6	10.0		30.0		40.0	60.0	0.0
3	7	10.0		30.0		40.0	60.0	0.0
4	8	10.0		30.0		40.0	60.0	0.0
5	6	10.0		30.0		40.0	60.0	0.0
6	8	10.0		30.0		40.0	60.0	0.0
8	7	10.0		30.0		40.0	60.0	0.0
7	5	10.0		30.0		40.0	60.0	0.0
1								
1	1	1	1	1	1			
2								
1	1	1	1	1	1			
3								
1	1	1	1	1	1			
4								
1	1	1	1	1	1			
2	2							
5								
5.0		0.0		0.0		0.0	0.0	0.0
7								
15.0		0.0		0.0		0.0	0.0	0.0
5								
0.0		30.0		0.0		0.0	0.0	1800.0
0.0		30.0		0.0		0.0	0.0	-1800.0
7								
0.0		15.0		0.0		0.0	0.0	900.0
0.0		15.0		0.0		0.0	0.0	-900.0

TABLE XVI

OUTPUT DATA FOR SPACE FRAME

$JOB BEV EDWARDS 2509-40040

ANALYSIS OF SPACE FRAMES

STRUCTURE DATA

M	N	NJ	NR	NRJ	E	G
8	24	8	24	4	29000.000	12000.000

COORDINATES OF JOINTS

JOINT	X	Y	Z
1	0.000	0.000	0.000
2	360.000	0.000	0.000
3	0.000	0.000	-240.000
4	360.000	0.000	-240.000
5	0.000	144.000	0.000
6	360.000	144.000	0.000
7	0.000	144.000	-240.000
8	360.000	144.000	-240.000

MEMBER DESIGNATIONS, PROPERTIES, AND ORIENTATIONS

MEMBER	JJ	JK	AX	IX	IY	IZ
1	1	5	10.000	30.000	40.000	60.000
AA		L(I)	CX	CY	CZ	
0.000		144.000	0.000	1.000	0.000	
MEMBER	JJ	JK	AX	IX	IY	IZ
3	3	7	10.000	30.000	40.000	60.000
AA		L(I)	CX	CY	CZ	
0.000		144.000	0.000	1.000	-0.000	
MEMBER	JJ	JK	AX	IX	IY	IZ
4	4	8	10.000	30.000	40.000	60.000
AA		L(I)	CX	CY	CZ	
0.000		144.000	0.000	1.000	-0.000	
MEMBER	JJ	JK	AX	IX	IY	IZ
5	5	6	10.000	30.000	40.000	60.000
AA		L(I)	CX	CY	CZ	
0.000		360.000	1.000	0.000	0.000	
MEMBER	JJ	JK	AX	IX	IY	IZ
6	6	8	10.000	30.000	40.000	60.000
AA		L(I)	CX	CY	CZ	
0.000		240.000	0.000	0.000	-1.000	
MEMBER	JJ	JK	AX	IX	IY	IZ
7	8	7	10.000	30.000	40.000	60.000
AA		L(I)	CX	CY	CZ	
0.000		360.000	-1.000	0.000	-0.000	
MEMBER	JJ	JK	AX	IX	IY	IZ
8	7	5	10.000	30.000	40.000	60.000
AA		L(I)	CX	CY	CZ	
0.000		240.000	0.000	0.000	1.000	

```
JOINT RESTRAINTS
JOINT      XTR       YTR       ZTR       XR       YR       ZR
    1        1         1         1          1        1       1
    2        1         1         1          1        1       1
    3        1         1         1          1        1       1
    4        1         1         1          1        1       1

LOAD DATA
 NLJ     NLM
  2       2
ACTIONS APPLIED AT JOINTS
JOINT      X FORCE      Y FORCE      Z FORCE
    5        5.000        0.000        0.000       0.000       0.000       0.000
    7       15.000        0.000        0.000       0.000       0.000       0.000
ACTIONS AT ENDS OF RESTRAINED MEMBERS DUE TO LOADS
MEMBER      AML1        AML2         AML3       AML4       AML5
    5        0.000      30.000        0.000      0.000      0.000    180.000
    AML7        AML8        AML9         AML10      AML11
     0.000      30.000      0.000        0.000      0.000-1800.000
MEMBER      AML1        AML2         AML3       AML4       AML5
    7        0.000      15.000        0.000      0.000      0.000     900.000
    AML7        AML8        AML9         AML10      AML11
     0.000      15.000      0.000        0.000      0.000    -900.000

JOINT DISPLACEMENTS AND SUPPORT REACTIONS
JOINT     XTR      YTR      ZTR      XR      YR      ZR
    1       0.000          0.000          0.000      0.000       0.000
  X FORCE        Y FORCE        Z FORCE
                                              X MOM       Y MOM      Z MOM
  12.476         29.011        0.247      20.264       6.813    -461.688
JOINT   XTR   YTR    ZTR XR    YR     ZR
    2     0.000       0.000       0.000      0.000      0.000      0.000
  X FORCE     Y FORCE     Z FORCE
                                          X MOM      Y MOM      Z MOM
  -18.331      30.985     -0.247   -20.686      6.837   1013.326
JOINT   XTR   YTR    ZTR XR    YR     ZR
    3     0.000       0.000       0.000     0.000      0.000      0.000
   X FORCE    Y FORCE    Z FORCE
                                          X MOM      Y MOM      Z MOM
     0.907     13.166     0.247     20.264      6.813    274.607
JOINT   XTR   YTR    ZTR XR    YR     ZR
    4     0.000       0.000       0.000     0.000       0.000     0.000
   X FORCE    Y FORCE     Z FORCE
                                          X MOM      Y MOM      Z MOM
   -15.052     16.838     -0.247    -20.686      6.837   1037.421

JOINT   XTR   YTR    ZTR XR    YR     ZR
    5     0.817      -0.014      -0.075  -0.000      -0.003     -0.036
   X FORCE    Y FORCE     Z FORCE     X MOM      Y MOM      Z MOM
     0.000      0.000       0.000      0.000      0.000    0.000
```

JOINT	XTR	YTR	ZTR	XR	YR	ZR
6	0.795	-0.015	0.079	0.000	-0.003	0.025

X FORCE	Y FORCE	Z FORCE	X MOM	Y MOM	Z MOM
0.000	0.000	0.000	0.000	0.000	0.000

JOINT	XTR	YTR	ZTR	XR	YR	ZR
7	1.896	-0.007	-0.075	-0.000	-0.003	-0.028

X FORCE	Y FORCE	Z FORCE	X MOM	Y MOM	Z MOM
0.000	0.000	0.000	0.000	0.000	0.000

JOINT	XTR	YTR	ZTR	XR	YR	ZR
8	1.877	-0.008	0.079	0.000	-0.003	0.004

X FORCE	Y FORCE	Z FORCE	X MOM	Y MOM	Z MOM
0.000	0.000	0.000	0.000	0.000	0.000

MEMBER END ACTIONS

MEMBER 1

AM 1 = 29.060
AM 2 = -12.476
AM 3 = 0.247
AM 4 = 6.813
AM 5 = -20.264
AM 6 = -461.688
AM 7 = -29.011
AM 8 = 12.476
AM 9 = -0.247
AM10 = -6.813
AM 11 = -15.352
AM12 = -1334.814

MEMBER 2

AM 1 = 30.985
AM 2 = 18.331
AM 3 = -0.247
AM 4 = 6.837
AM 5 = 20.686
AM 6 = 1013.326
AM 7 = -30.985
AM 8 = -18.331
AM 9 = 0.247
AM10 = -6.837
AM11 = 14.930
AM12 = 1626.404

MEMBER 3

AM 1 =	13.166
AM 2 =	-0.907
AM 3 =	0.247
AM 4 =	6.813
AM 5 =	-20.264
AM 6 =	274.607
AM 7 =	-13.166
AM 8 =	0.907
AM 9 =	-0.247
AM10 =	-6.813
AM11 =	-15.352
AM12 =	-405.277

MEMBER 4

AM 1 =	16.838
AM 2 =	15.052
AM 3 =	-0.247
AM 4 =	6.837
AM 5 =	20.686
AM 6 =	1037.421
AM 7 =	-16.838
AM 8 =	-15.052
AM 9 =	0.247
AM10 =	-6.837
AM11 =	14.930
AM12 =	1130.027

MEMBER 5

AM 1 =	17.903
AM 2 =	29.134
AM 3 =	0.247
AM 4 =	-0.662
AM 5 =	-44.489
AM 6 =	1346.809
AM 7 =	-17.903
AM 8 =	30.866
AM 9 =	-0.247
AM10 =	0.662
AM11 =	-44.551
AM12 =	-1658.709

MEMBER 6

AM 1 =	0.000
AM 2 =	0.119
AM 3 =	-0.428
AM 4 =	-32.305

AM 5 =	51.388
AM 6 =	14.268
AM 7 =	-0.000
AM 8 =	-0.119
AM 9 =	0.428
AM10 =	32.305
AM11 =	51.388
AM12 =	14.268

MEMBER 7

AM 1 =	15.480
AM 2 =	16.957
AM 3 =	0.247
AM 4 =	-0.662
AM 5 =	-44.551
AM 6 =	1097.722
AM 7 =	-15.480
AM 8 =	13.043
AM 9 =	-0.247
AM10 =	0.662
AM11 =	-44.489
AM12 =	-393.282

MEMBER 8

AM 1 =	-0.000
AM 2 =	0.122
AM 3 =	-0.428
AM 4 =	11.994
AM 5 =	51.302
AM 6 =	14.690
AM 7 =	0.000
AM 8 =	-0.122
AM 9 =	0.428
AM10 =	-11.994
AM11 =	51.302
AM12 =	14.690

CHAPTER VI

SPECIAL PURPOSE COMPUTER PROGRAMS

The programs that have been presented are of a general nature, and they may be applied to structures in a broad category. The stiffness method programs will handle most framed structures. Occasionally, an engineer will find himself doing a certain set of calculations frequently enough to make the problem worth programming. This may simply involve programming a set of "plug-in" equations, or it may be extremely complicated. In any event, it may fall into the category of a special purpose problem, which can not be properly satisfied by a general purpose program, such as the stiffness method programs presented in the last chapter. Special purpose programs can be on an infinite number of small or special topics.

A few special purpose programs, which the author finds to be particularly handy, are demonstrated. They should lead the engineer to realize that many routine problems can be done more economically on a computer, if they re-occur often enough.

Computer Analysis and Design of Multi-Span Highway Bridges

Highway bridges have always been a somewhat repetitive operation, and engineers have specialized in bridge design alone. Most state highway departments have a bridge design section which does the same calculations over and over, for years on end. The analysis of simple span and continuous bridge structures is an algorithm, or chain of logical decisions, which can be programmed. In the period since about 1962, several state highway departments have written programs for the analysis and design of multi-span continuous bridges of variable cross section. The Georgia Highway Department wrote a program for the I.B.M. 1620, 60k, which has subsequently been adopted by the Oklahoma Highway Department. The Maine State Highway Commission published a series of bridge design programs in January, 1965, utilizing the method of moment distribution.[1] In 1963 the Wisconsin State Highway Commission, Bridge Section, published a group of nine programs entitled Continuous Beam Analysis and Steel Beam Design[2], which will be examined here.

In its original form, nine programs for bridge analysis and design were written by the engineers of the Wisconsin Highway Commission to compute the beam characteristics, dead load moments and shears, live load moments, shears, and reactions for a variable cross section continuous bridge of up to five spans, based on the 1961 AASHO specifications. The

[1] W. J. Verrill, R. L. Mallar, and D.R. Fields, Continuous Beam Series, (I.B.M. Users Group Program No. 9.2.053).

[2] James F. Gibbons and Stanley W. Woods, Continuous Beam Analysis and Steel Beam Design, (I.B.M. Users Group Program No. 9.2.017).

cover plates are then designed for a basic web section using AASHO alternating stress specifications and AWS specifications.

The program was originally written in nine consecutive passes to be run on the I.B.M. 1620, 20k, computer. All input was on paper tape, and only the paper tapes were released for publication, along with a discussion of the programs. The source program was not released, and paper tape feed 1620's soon became obsolete and unavailable. In short, the program was inoperable because the proper machine was not available to run the object tape, and the source program was not published. Often, authors are reluctant to release their programs for exploitation by others. The author of this report had extensive negotiations with the Wisconsin Highway Commission and finally succeeded in obtaining a "bugged listing" of the program from the authors. The program was filled with errors incurred when the actual source listing was typed by a secretary before being released. In addition to these errors, the programs could not be run on a standard 1620, 20k, because they overflowed the memory. They had been rewritten in machine language by the original authors to counteract this situation. The author of this report spent seven months debugging the nine programs and combining them in one very large program to be run on the I.B.M. 7040. The nine programs were "chained" together (a special feature of the 7040 compiler) to run in series, one at a time. The output of a program is written an magnetic tape for temporary storage, the next program is read in, and the data is read off of the magnetic tape. In this manner, each of the programs is called into memory by the computer at the proper time, operated on, data is written on tape or stored in a COMMON statement, the program exits the memory, the next program in the series is called into memory, data is read from cards, tape, or COMMON, etc. - until all nine programs have been run in succession. All of these problems are mentioned to make the reader aware of a few of the many problems encountered in trying to run someone else's program. This points to the advantage of writing your own programs. "Canned programs" rarely work without some alterations. The precise manipulations involved in many areas of this program are a mystery to all but the original author, for in many cases there are no comment statements to explain what is being done. These are a few of the disadvantages of this program.

The strong advantages of this program make it worth all of the trouble. It takes the I.B.M. 7040 computer about one and one-half minutes to do all of the calculations for the analysis (and design of cover plates) of a five span continuous, parabolically haunched plate girder under AASHO H20-S16 truck loading. This makes it possible to try many alternatives for the design of a particular bridge, in search of the best design. The cost of running a proposed design on the computer is probably in the \$15 - \$20 range. It would be impossible for an engineer to do the calculations this economically in a commercial operation. The author of this report has run four bridges in a row with only seven minutes of execution time used on the 7040. The client can certainly save money and materials by having the engineer do the calculations on the computer. If nothing else, the computer offers an excellent check on manual calculations.

A consulting engineer who does not do bridge design exclusively can, in effect, store his knowledge of bridge design in a computer program for instant recall when a bridge job comes along. Likewise, an engineer who is not completely versed in the design of bridges can draw on the knowledge of an expert bridge designer who has stored his knowledge in this program. Briefly, one does not have to be as polished as a professional bridge designer to do the calculations involved in the analysis of a large bridge, with the aid of this program. This program is not a substitute for sound engineering judgment, however.

The words of the original authors (James F. Gibbons and Stanley W. Woods, Continuous Beam Analysis and Steel Beam Design), best describe the basic operation of the program.

> "The beam characteristics are computed first, based on the cross section data given as input. There are two types of data that are possible.
>
> Type I is used when the moment of inertia variation is known. This is commonly used with wide flange sections, and could be used as a first step in plate girder design.
>
> Type II is used when the exact description of the section is given, which is usually the case for plate girders and concrete beams. The input section consists of the web and plate sizes. The web may be straight or have straight or second degree parabolic haunches.
>
> The concentrated angle change method of describing the elastic slope of a beam is used to obtain fixed end moments, carry over factors and relative stiffness (If the actual section is used, the absolute stiffness is found, except for the modulus of elasticity).
>
> The span is broken into twenty segments for this method. If there are hinges in the span, the beam characteristics are modified using a method outlined by PCA[3].
>
> There may be three variations of uniform dead load in each span. The dead load moments are found by the slope deflection method. Equations are set up for each span and placed in matrix form. The coefficient

[3] "Beam Factors and Moment Coefficients for Members with Intermediate Expansion Hinges, (PCA Bulletin ST 75), 1948.

matrix is inverted and the fixed-end moments are inserted to solve for the joint rotations, and subsequently the dead load end moments. Simple beam moments are superimposed on the end moments and the final dead load moments are computed at tenth points. The dead load shears are computed along with the dead load moments.

In the category of live load moments, an end moment influence line is computed for each span. This influence line is based on the variable moment of inertia entered as input, or a constant moment of inertia, depending on the designer's choice.

Using the end moment influence lines, an influence line for each 10th point in the span is computed with ordinates on the influence line computed at the 20th points. Maximum positive, maximum negative, and total areas are computed for the influence lines to determine curb and sidewalk moments and maximum positive and negative live load lane moments.

Maximum positive and negative truck moments are then computed. For positive moment, a concentrated load is placed at the point being considered for positive moment and the remaining wheels placed to provide maximum moment. For negative moment, the concentrated load is placed at the maximum negative ordinate and the other wheels placed to provide maximum moment. AASHO specifies the wheel spacing to be 14 feet, with the trailer axle being allowed to vary between 14 and 30 feet. The truck is placed in all combinations of allowable positions to determine the maximum moments.

The maximum positive truck moment is compared to the maximum positive lane moment, and the maximum value is printed. This is repeated for the negative moments. The moments that are printed have been multiplied by impact and distribution factors.

Live load shears are found in a way similar to live load moments, by computing a shear influence line in place of a moment influence line.

Reactions for curb and sidewalk loads are the sums of the shears at the supports.

Reactions are computed for both lane and truck loads in two ways. First, they are found for one lane and one truck, with no impact or distribution included. Then, they are found by multiplying the results by the distribution factor (LLR) and the average impact factor for the first and second spans. LLR is used only on the load at the support. The other loads and also the shears use the moment distribution factor.

A steel beam may be designed for one to three basic allowable stresses, compositely in the positive moment regions and non-compositely throughout.

Some assumptions are made so that the approximate thicknesses of cover plates are used. If the depth is less than 37 inches, a wide flange section is assumed, and the width of the cover plate is assumed to be 10 inches. If the depth is greater than 37 inches, a plate girder section is assumed and the width of cover plate is assumed to be 16 inches.

Cover plates are determined to be added to the basic web section. The basic web sections for plate girders is the actual web, and for wide flange beams it is the beam. Cover plate sizes are determined using the allowable stresses. In the non-composite zone, the sizes are based on the allowable basic, AASHO, and AWS stresses.

The allowable AASHO stresses are based on the ratio of minimum to maximum stresses and specified group loading condition. AWS allowable stresses are computed similarly and single lane loading is applied where applicable, using the ratio of distribution factors for the live load moments. For wide flange beams, the formulas for fillet welds are used and for plate girders the formulas for A36 steel butt welds are used.

If the total positive moment is greater than the total negative moment, a composite design is made. The dead load is placed on the steel section and the sidewalk and live loads are placed on the composite section. Stresses in the top and bottom plates and the concrete are checked against the allowable stresses. Plate areas are added where needed until the stresses in the section are within the allowable.

> A comparison of basic and AASHO alternating allowable stresses is made, and the smaller is used for designing the section. Plate sizes also are determined by the AWS specifications to help determine appropriate cut-off points.
>
> A minimum area for the top plate is part of the input, because a plate girder with composite action may not require a top cover plate, but a plate would be necessary for lateral stability and to hold shear connectors.
>
> Shear connector spacings are computed at the 10th points along the span, using the maximum value of composite moment of inertia in the span."

A general flow chart will show the basic sequence of operations in the program.

LINK 1
Read in numbers of spans, type of loading (H20-S16 etc.), whether or not live load is on constant or variable section I, simple beam distribution factor at support for determining reactions, sidewalk live load (lb./ft), dead load of curb lb./ft.), length of spans, location of expansion hinges, uniform dead load (3 possible variations/span), live load distribution factor according to AASHO, number of allowable stresses to be used in the design (3 max.), allowable stresses, allowable concrete stress, effective width of slab, slab thickness, distance from the top of the web to the bottom of the slab, minimum area of top cover plate for composite design, resisting force of one shear connector, modular ratio of steel to concrete, AASHO group number for determining allowable stresses from alternating loads, dimensions and properties of wide flange or plate girder section at all points in each span (haunches included).

Compute the moment of inertia of a plate girder from the dimensions given. Compute the relative stiffnesses, carry-over factors and fixed-end moments.

↓

LINK 2.
Modify the beam characteristics for the presence of expansion hinges.

↓

LINK 3
This is a linkage program which was used in the 1620 version to transfer the beam characteristics computed in LINK 2 to different addresses in order to make maximum use of the 20k memory of the 1620. It was simpler to leave it in the 7040 version than to take it out. It also prints out the headings for the dead load moments and shears.

↓

LINK 4
This is the slope-deflection solution program. This program sets up simultaneous linear equations and solves them for dead load moments and shears, and for pier moment influence lines. The dead load moments are stored on tape (4), to be used in the steel design program.

↓

Figure 32.

Flow Chart of Bridge Analysis and Design

LINK 5
This is another linkage program which was used on the 1620 which stores the pier moment influence lines, so as to use one-half the storage that they previously took. It was easier to leave it in the 7040 version than to take it out. This program also prints out live load moment headings.

LINK 6
This is the live load moment program. An influence line is computed and lane and truck loads are applied to find the maximum positive and negative moments in a structure.

LINK 7
This program is the live load shear program, which computes live load shears at 10th points along the span and also reactions at the supports.

LINK 8
This is another linkage program which does no calculations, but only rearranges storage.

LINK 9
As discussed previously, the steel beam is designed. Cover plate is added and subtracted to the basic section, both compositely and non-compositely. Shear connector spacings also are computed.

Figure 33.

Continuation of Bridge Flow Chart

This should give the reader a very general idea of what the program does. To explain the program in any great detail would take one hundred pages, perhaps. The only people who fully understand the interworkings of the program are the original authors. Comment statements are very scarce in the source listing, and it is often difficult to determine what is being done at various stages. This program is presented to give the reader a taste of what is availabe in the field of bridge programming. For a fuller discussion of the program, one should refer to the I.B.M. Users Group literature published on it. (see footnote 2, p. 103).

This program is capable of handling wide flange, plate girder, and concrete continuous beams, of variable cross section. A two span continuous parabolically haunched plate girder bridge has been run as a sample problem. Sense switch 3 is turned on to run a plate girder or concrete section, and off for a wide flange section. The input data for the sample problem is illustrated on the next page.

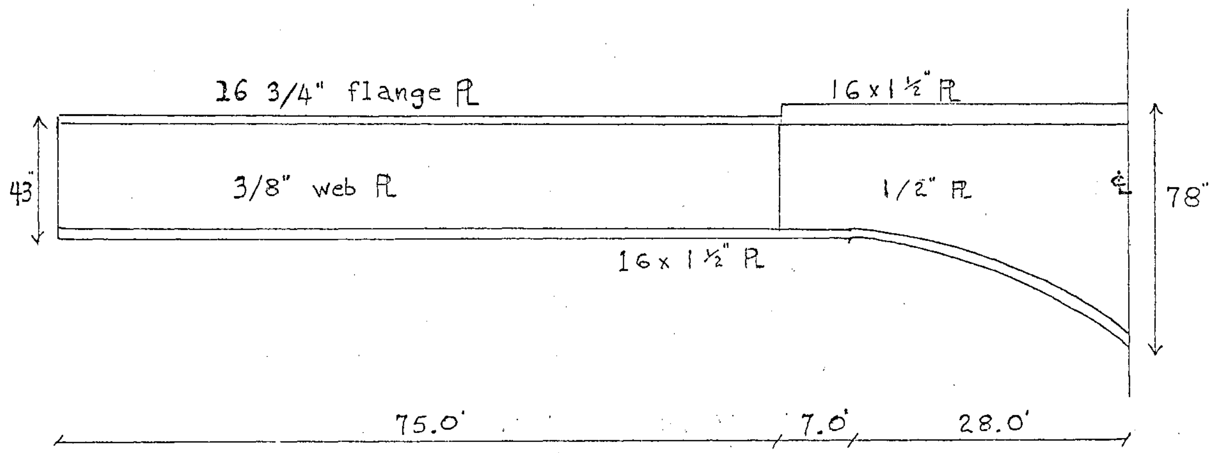


Figure 34.

Two Span Continuous Plate Girder

H20-S16 Truck loading

The simple beam distribution factor at the support of the beam, for determining the reactions, is 1,833.

Live load on the sidewalk is 0.0 lb./ft.

Curb weight is 0.0 lb./ft.

The AASHO live load distribution factor is 1,636

Dead load weight of slab beam hardware is 1080.0 lb./ft.

No hinges are present.

Allowable steel stresses of 20.0 and 25.0 ksi. are tried.

Allowable concrete stress is 1.40 ksi.

Effective slab width is 84.0".

Effective slab thickness is 7".

Shear connector strength is 23.8 kips.

Modular ratio of steel to concrete is 10.0.

Minimum area of top cover plate is 12.0 sq. in.

AASHO group number for determining allowable stresses from alternating loads is 2.

TABLE XVII

INPUT DATA FOR BRIDGE

1

1620.000	2.000	4.000	0.000	1.833	0.000	0.000	
1.000	1.000	1.000	-1.000				
110.000	1.646	1080.000					
1.000	82.000	43.000	43.000				
1.000	110.000	-43.000	78.000				
2.000	75.000	16.000	.750				
2.000	110.000	16.000	1.500				
3.000	110.000	16.000	1.500				
4.000	75.000	1.000	.375				
4.000	110.000	99999.999	.500				
99999.000	1.646	1080.000					
0.000	0.000	0.000	75.000	0.000	40.000		
0.000	0.000	35.000	0.000	40.000	0.000		
2.0	20.0	25.0	0.0	1.4	84.0	7.0	0.0
12.0	23.8	10.0	2.0				

TABLE XVIII

OUTPUT DATA FOR BRIDGE

DL MOM(FT-KIPS) AND SHEAR(KIPS) AT X/20TH PTS

X	MOM.	SHEAR
SPAN 1		
0	0.0	39.9
2	374.1	28.1
4	617.5	16.2
6	730.3	4.3
8	712.4	-7.6
10	563.7	-19.5
12	284.5	-31.3
14	-125.6	-43.3
16	-669.5	-55.6
18	-1349.0	-67.9
20	-2164.0	-80.3

MAX. LIVE LOAD MOMENTS AT X/20 PTS.-IN FT. KIPS

X	CURB +(+)SDK.LL	CURB+(-)SDK.LL	+LL	+LL
SPAN 1				
2.0	0.0	-0.0	592.5	-97.6
4.0	0.0	-0.0	984.1	-195.3
6.0	0.0	-0.0	1197.8	-292.9
8.0	0.0	-0.0	1268.9	-390.6
10.0	0.0	-0.0	1215.5	-488.2
12.0	0.0	-0.0	1056.8	-585.9
14.0	0.0	-0.0	803.4	-683.5
16.0	0.0	-0.0	486.6	-781.2
18.0	0.0	-0.0	130.0	-1182.9
20.0	0.0	-0.0	0.0	-1766.5

LIVE LOAD SHEARS AND REACTIONS

LEFT SUPPORT REACTIONS

PIER-LANE	BRG-LANE	PIER-TRUCK	BRG-TRUCK
SPAN 1			
55.4	58.1	63.8	67.1

SHEARS AT X/20TH PTS

X	SDK.+CURB	MAX. LL
0.0	0.0	63.3
2.0	0.0	53.9
4.0	0.0	44.7
6.0	0.0	36.1
8.0	-0.0	-29.4
10.0	-0.0	-37.9
12.0	-0.0	-45.6
14.0	-0.0	-52.6
16.0	-0.0	-58.6
18.0	-0.0	-64.0
20.0	-0.0	-72.2

SPAN 2
119.5 121.6 71.6 74.8
CONTINUOUS STEEL BEAM DESIGN AT X/10TH POINTS

	NON-COMPOSITE			COMPOSITE FS AND AASHO		AWS	
FS	A-FS	A-AASHO	A-AWS	A-BTM	A-TOP	A-BTM	A-TOP
SPAN 1							
1							
20.0	10.9	8.9	8.1	8.8	12.0	8.8	12.0
25.0	8.1	8.9	8.0	6.4	12.0	6.4	12.0
2							
20.0	19.8	16.8	15.4	18.2	12.0	18.2	12.0
25.0	15.2	16.8	15.3	14.6	12.0	14.6	12.0
3							
20.0	24.4	21.3	19.5	23.0	12.0	23.0	12.0
25.0	18.9	21.2	19.4	19.4	12.0	19.4	12.0
4							
20.0	25.1	22.9	20.8	23.8	12.0	23.8	12.0
25.0	19.5	22.8	20.7	21.0	12.0	21.0	12.0
5							
20.0	22.3	21.8	19.6	20.4	12.0	18.6	12.0
25.0	17.2	21.7	19.5	19.8	12.0	19.8	12.0
6							
20.0	16.1	18.3	16.1	13.6	12.0	13.8	12.0
25.0	12.3	18.2	16.0	16.2	12.0	16.2	12.0
7							
20.0	7.8	12.6	10.7				
25.0	5.5	12.6	10.7				
8							
20.0	15.9	14.7	13.4				
25.0	12.0	14.7	13.4				
9							
20.0	22.7	16.1	15.2				
25.0	17.2	16.1	15.2				
10							
20.0	24.0	15.6	14.8				
25.0	17.9	15.6	14.8				

SHEAR CONNECTOR SPACINGS (IN.)

0.0	19.9
1.0	23.4
2.0	28.2
3.0	34.9
4.0	42.9
5.0	33.3
6.0	27.6
7.0	24.0
8.0	21.5
9.0	19.7
10.0	17.5

Gable Frame Analysis

A common frame analysis problem is the gable frame. This problem could be done with the stiffness method plane frame program, but not on a small 1620 computer. A volume of condensed solutions to common frame problems is Frames and Arches, by Valerian Leontovich. It contains plug-in algebraic formulas for the solutions to portal frames, trapezoidal frames, gable frames, arches, etc. These formulas are easily programmed on the 1620 to provide a quick and comprehensive solution to a routine problem. The formulas for the analysis of a pinned base gable frame, with haunched members optional, have been programmed. It gives a solution for all of the support reactions and bending moments due to uniform load on the roof, snow drift load on the reef, and wind load.[1]

It is necessary to use this book[1] in conjunction with the operation of the program to obtain the values of the data used as input for the solution of the formulas. All input data is explained in the comment statements of the source program. A sample gable frame with haunched members has been run, and it is presented on the next page.

[1] Valerian Leontovich, Frames and Arches, (McGraw-Hill Book Company, 1959,) p. 295.

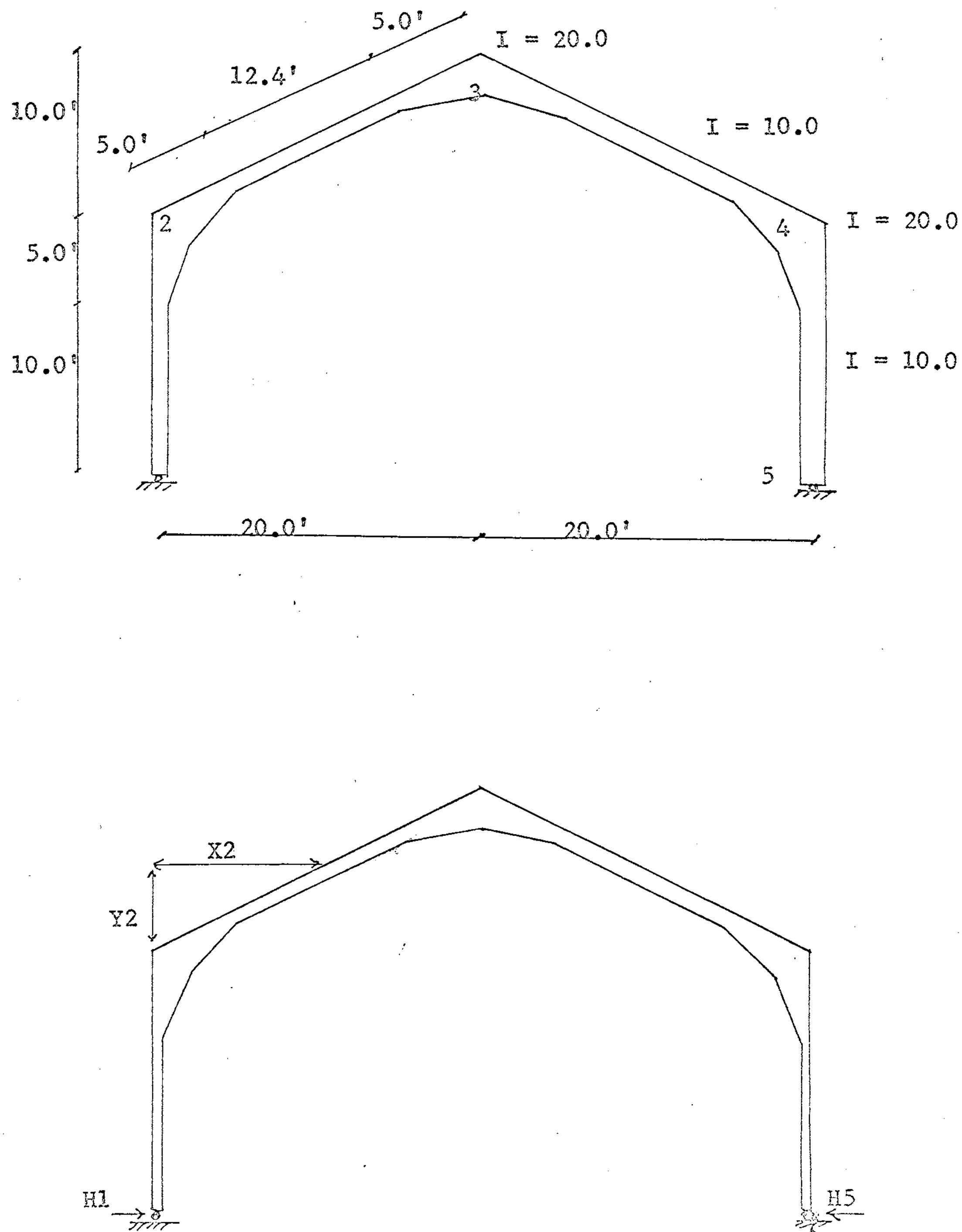


Figure 35.

Sample Gable Frame Problem

TABLE XIX

INPUT DATA FOR GABLE FRAME

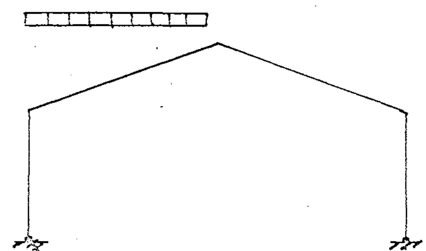
.49	.45	.47	.47				
3.95	3.10	3.3	3.3	1.85	1.9	15.0	10.0
22.4	40.0	10.0	10.0	2.0	2.0	1.0	1.0

OUTPUT DATA

LEFT BAY HAS UNIFORM VERT LOAD = 2.000 KIPS/FT

HI = H5	M2 = M4	M3	V1	V5
6.327	-94.910	41.816	30.000	10.000

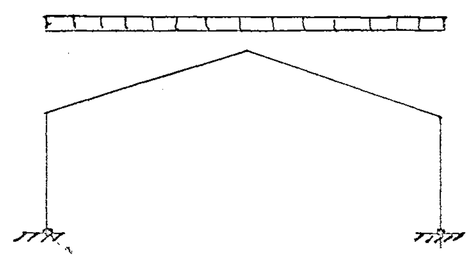
X2	MX2
2.000	-45.237
4.000	-3.564
6.000	30.107
8.000	55.780
10.000	73.453
12.000	83.125
14.000	84.798
16.000	78.471
18.000	64.143
20.000	41.816



BOTH BAYS HAVE UNIFORM VERT LOAD = 2.000 KIPS/FT

H1 = H5	M2 = M4	M3	VI	V5
12.654	-189.820	83.632	40.000	40.000

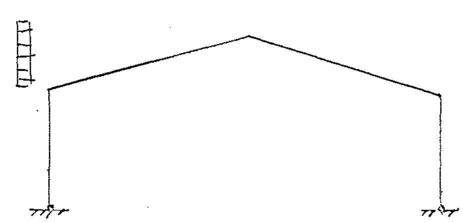
X2	MX2
2.000	-126.474
4.000	-71.129
6.000	-23.784
8.000	15.561
10.000	46.906
12.000	70.251
14.000	85.597
16.000	92.942
18.000	92.287
30.000	83.632



UNIFORM WIND LOAD = 1.000KIPS/FT ON ROOF ONLY

H1	H5	M2	M3	M4	V1	V5
-5.390	4.609	80.860	-15.233	-69.139	-4.999	4.999

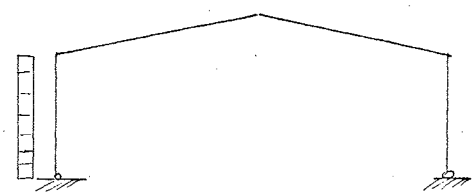
Y2	MY2
1.000	75.750
2.000	69.641
3.000	62.532
4.000	54.422
5.000	45.313
6.000	35.204
7.000	24.094
8.000	11.985
9.000	-1.123
10.000	-15.233



UNIFORM WIND LOAD= 1.000 KIPS/FT ON COLUMNS

HI	H5	M2	M3	M4	VI	V5
-12.027	2.972	67.918	-18.051	-44.581	-2.812	2.812

Y2	MY2
1.500	16.916
3.000	31.583
4.500	44.000
6.000	54.167
7.500	62.084
9.000	67.751
10.500	71.168
12.000	72.335
13.500	72.252
15.000	67.918



ANALYSIS OF PRESTRESSED CONCRETE SECTIONS

The process of designing a prestressed concrete cross section for any applied system of loads is a highly repetitive procedure of trial and error. The initial trial section may be based on an educated guess derived from formulas such as the following formulas presented in Chapter 6 of Design of Prestressed Concrete Structures, by T. Y. Lin.[1]

$$F = M_T/.65\ h \qquad h = k\sqrt{M} \qquad A_c = F/.50 f_c$$

where h = depth of beam in inches

M = maximum bending moment in ft-kips

k = a coefficient varying from 1.5 to 2.0

F = final prestress force

M_T = total moment acting on the section (D.L. + L.L.)

A_c = required area of section

f_c = maximum allowable compression stress at working load.

From these crude values a trial section is assumed and the analysis is made. This trial and error process suggests itself well for computer application. If, from the properties or dimensions of a general section, a computer analysis can be performed, much of the repetitious work of a design problem can be accelerated. Just as many trial sections may be required as by manual methods, but they may be done in a matter of a few minutes, rather than a much longer time by long hand. As an example, seven trial sections were checked for a given beam in one-half hour with

[1] T.Y. Lin, Design of Prestressed Concrete Structures, (John Wylie & Sons, 1955).

the aid of the computer. This included card punching time between each new revised trial section.

In searching for a ready made computer program for the analysis of prestressed concrete sections, one encounters the usual difficulty in trying to get something for nothing. Many authors in the field of structural engineering love to give flowery but vague descriptions of how to apply the computer to the analysis of various types of structures. Very few authors give print-outs of their actual source programs. Until the recent publication of some texts dealing with structural programming, one's only recourse was the I.B.M. Users Group Library of Structural Programs, or starting from scratch.

Several articles on the subject of programming of prestressed concrete have appeared in the Journal of the Prestressed Concrete Institute.[1,2] As usual, these were nothing but a big pep talk about computer programming of prestressed concrete. No programs were given. The first article referred to did state that the basis of the material contained in the program (which was not presented) being discussed, was contained in <u>Design of Prestressed Concrete Structures</u>, by T. Y. Lin. (See Foot note 1, P. 132).

Reverting back to this original source (T. Y. Lin), a program has been written by the author, and it is presented in this report. It was written from scratch, largely based on the material contained in Chapter

[2] Peter C. Patton and Harold R. Hutchens, <u>Designing Prestressed Concrete Slabs with a Digital Computer</u>, (P.C.I. Journal, June, 1962).

[3] A. D. St. John, <u>Computer Design of Prestressed Concrete</u>, (P.C.I. Journal, August, 1963).

6 of Lin's text. It is written for the I.B.M. 1620 computer, but it may easily be modified to run on other computers using basic Fortran. The author assumes that the reader has a fundamental knowledge of basic Fortran.

In writing this program, it was desired to incorporate as wide a range of prestressed sections as possible. Relying on the basic "I" shaped section as being the most general, it is possible to vary the depth of the beam and the size of the top and bottom flanges (including fillets) In fact, the dimensions of the bottom flange may be reduced to zero to form a "T" shaped section. As an added option, the dimensions of a composite section may be read in (including the slab), and the analysis may be done on a composite section. The composite option is manipulated by sense switch 3, as will be shown later.

The question of allowing or not allowing tension in the section is taken into consideration. From the values of the strength of the concrete at the time of initial prestress and at 28-day strength (as read into the computer), the A.C.I. allowable stresses are computed for compression and tension. If tension is not desired, or if a different value of tension than the A.C.I. allowable is desired, by turning on sense switch 4, seperate values of tension may be read in. If no tension is desired, simply read in the new value of tension as 0.0.

The options of analyzing any basic "I" shaped section for composite or non-composite action, with or without the consideration of tension, make the program very versatile. In reference to the variations in the shape of the sections that are possible for this computer analysis, a few possible shapes are shown below.

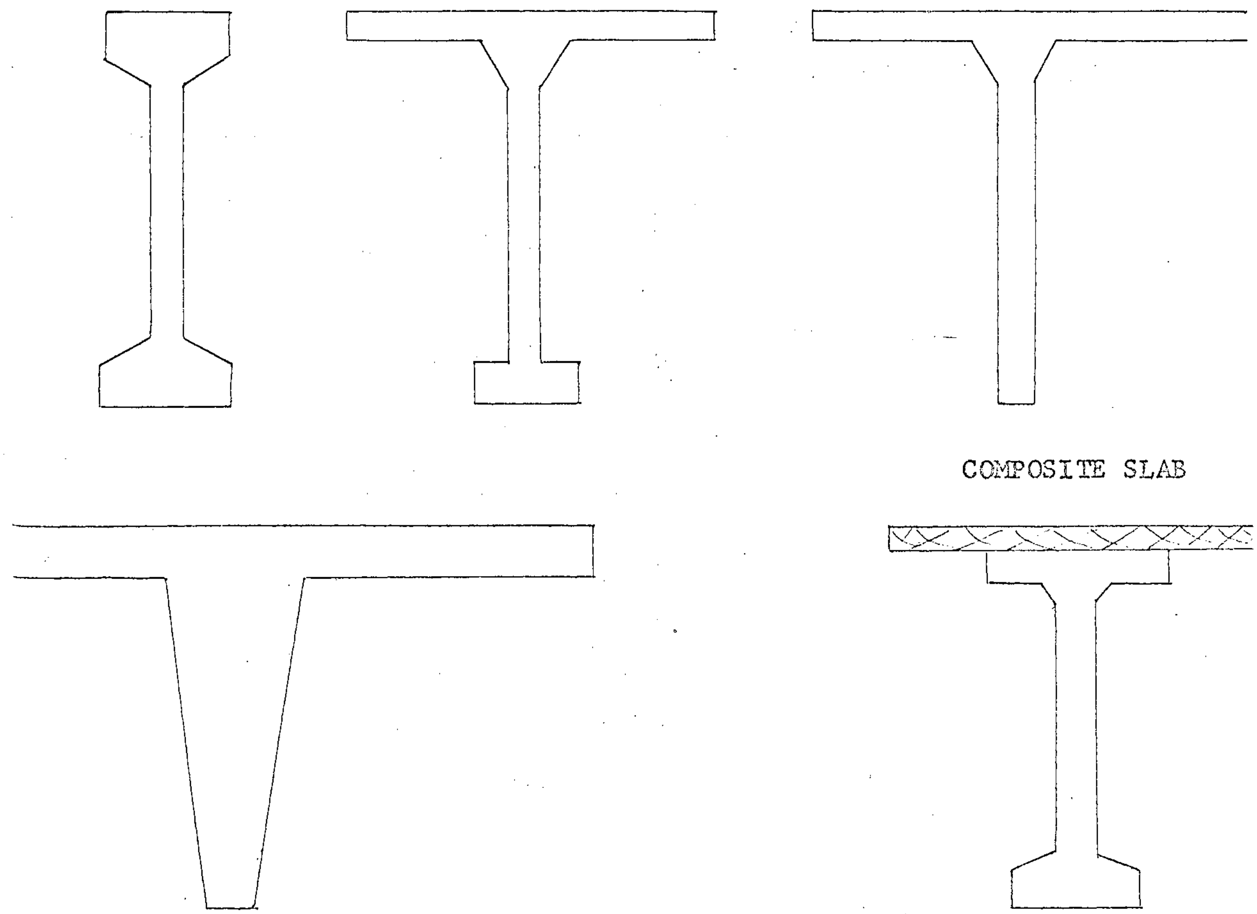


Figure 36.

Possible Variations in Shape of Section

The basic dimensions of the beam (in inches) to be read into the computer are shown in Figure 37, and they are further explained in the initial comment statements at the start of the source program.

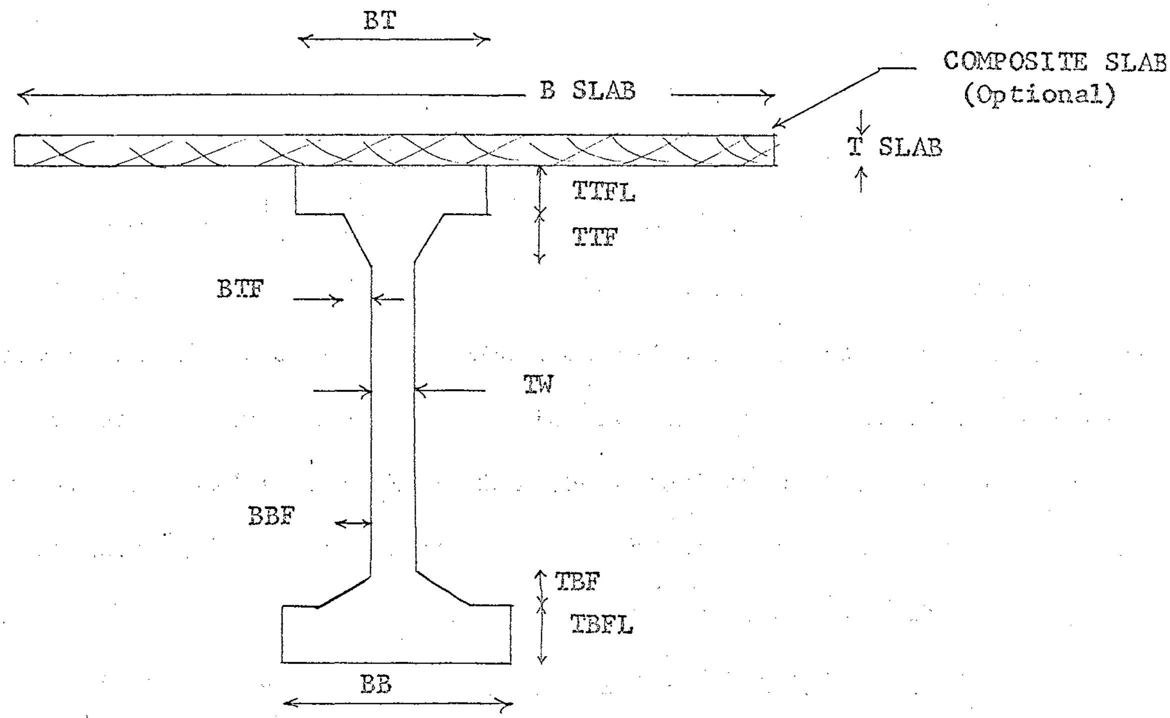


Figure 37.

Dimensions of Section

In addition, the values of dead load girder moment, live load girder moment, and composite slab dead load moment (optional) are read in as XMG, XML, and XMS respectively. All bending moments are in inch-kips. Composite slab dead load moment XMS and composite slab dimensions (BSLAB and TSLAB) are read in as an option if sense switch 3 is on. The initial prestress force FO, the percent loss of prestress PL, the initial compression strength of the concrete at the time of prestress, FC1, and the 28-day strength FC2, are read into the computer. Finally, the allowable tension in concrete at time of prestress, FC5, and the allowable tension under working load, FC6, which differ from the A.C.I. allowable tension, may be read in as an option by turning on sense switch 4. All of the input data is thoroughly defined in the comment statements at the start of the source program.

The first portion of this program is devoted to finding the properties of the section such as the area, location of the neutral axis, location of the kern points, moment of inertia, and required eccentricity. The A.C.I. allowable stresses in tension and compression are computed from f and f read in. Then the required areas of the section are computed for the tension and compression stresses to be within the allowable. If this required area is less than the actual area, then the section is satisfactory, but a smaller section may be found by the trial and error process. The method of finding these required areas is taken from articles 6-6 and 6-7 of <u>Design of Prestressed Concrete Structures</u>, by T. Y. Lin. (See footnote1 , p.132).

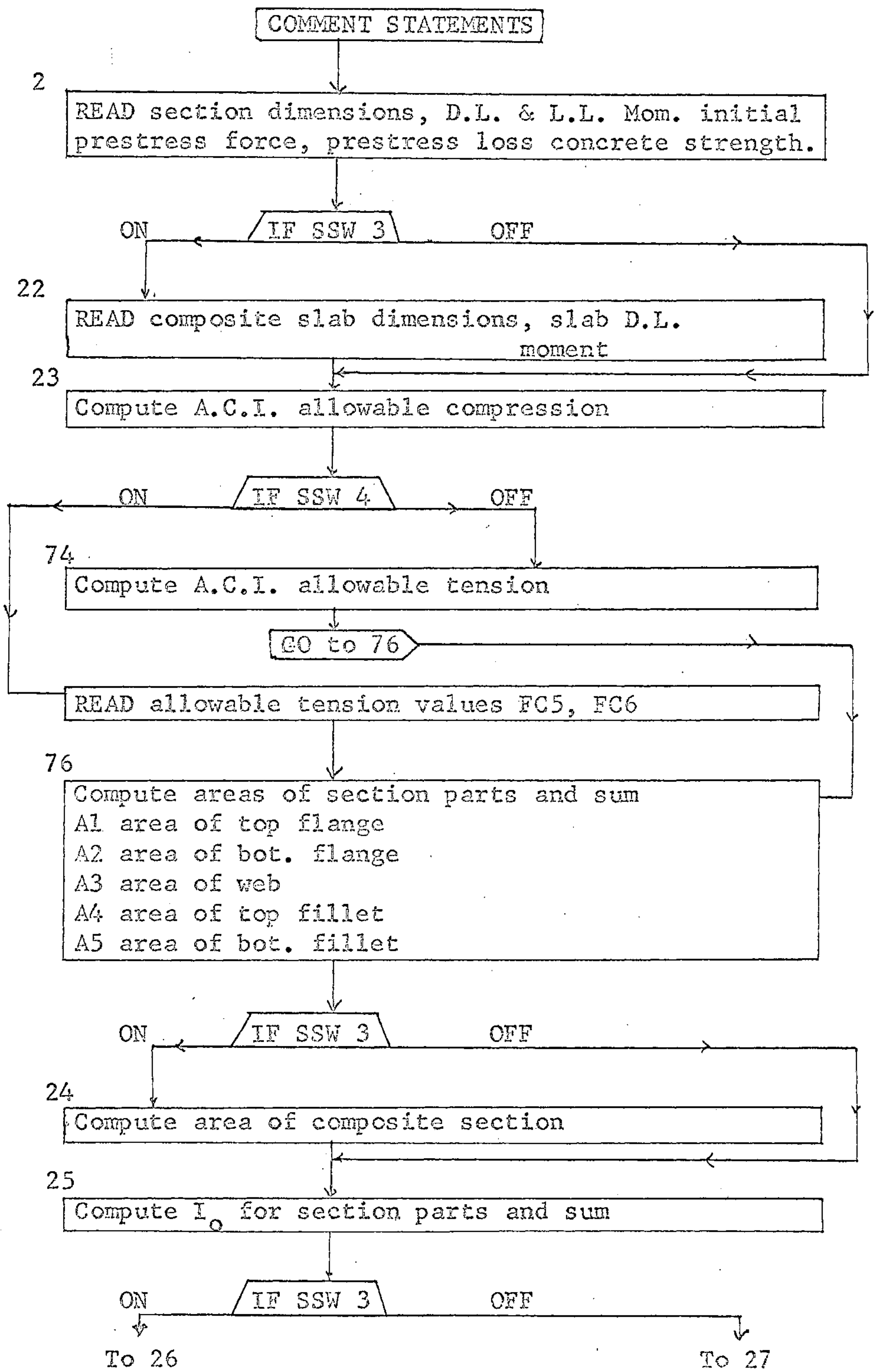


Figure 38.

Flow Chart for the Analysis of
Prestressed Concrete Single "T", "I", or Composite Sections

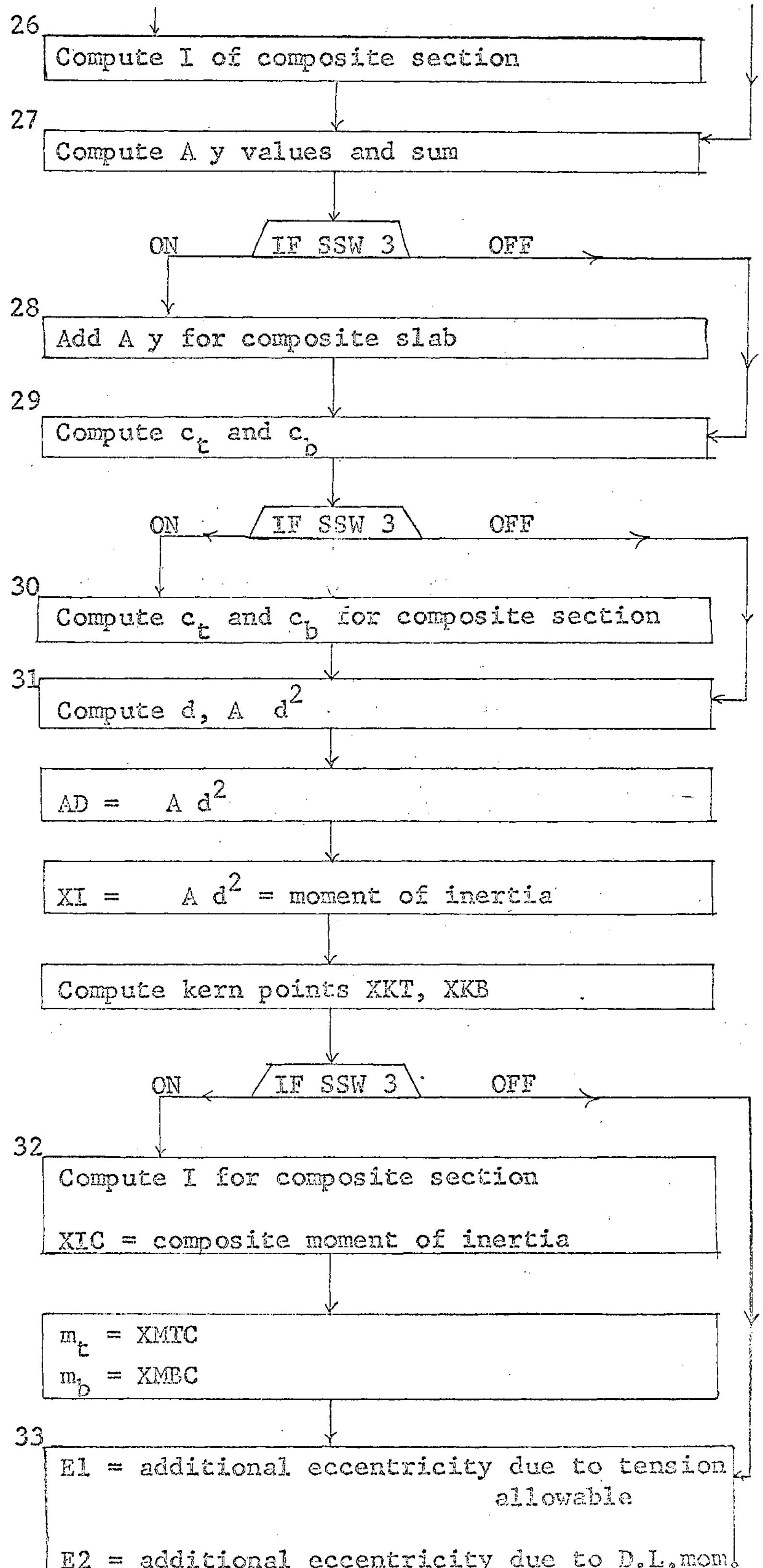


Figure 39. Continuation of Flow Chart

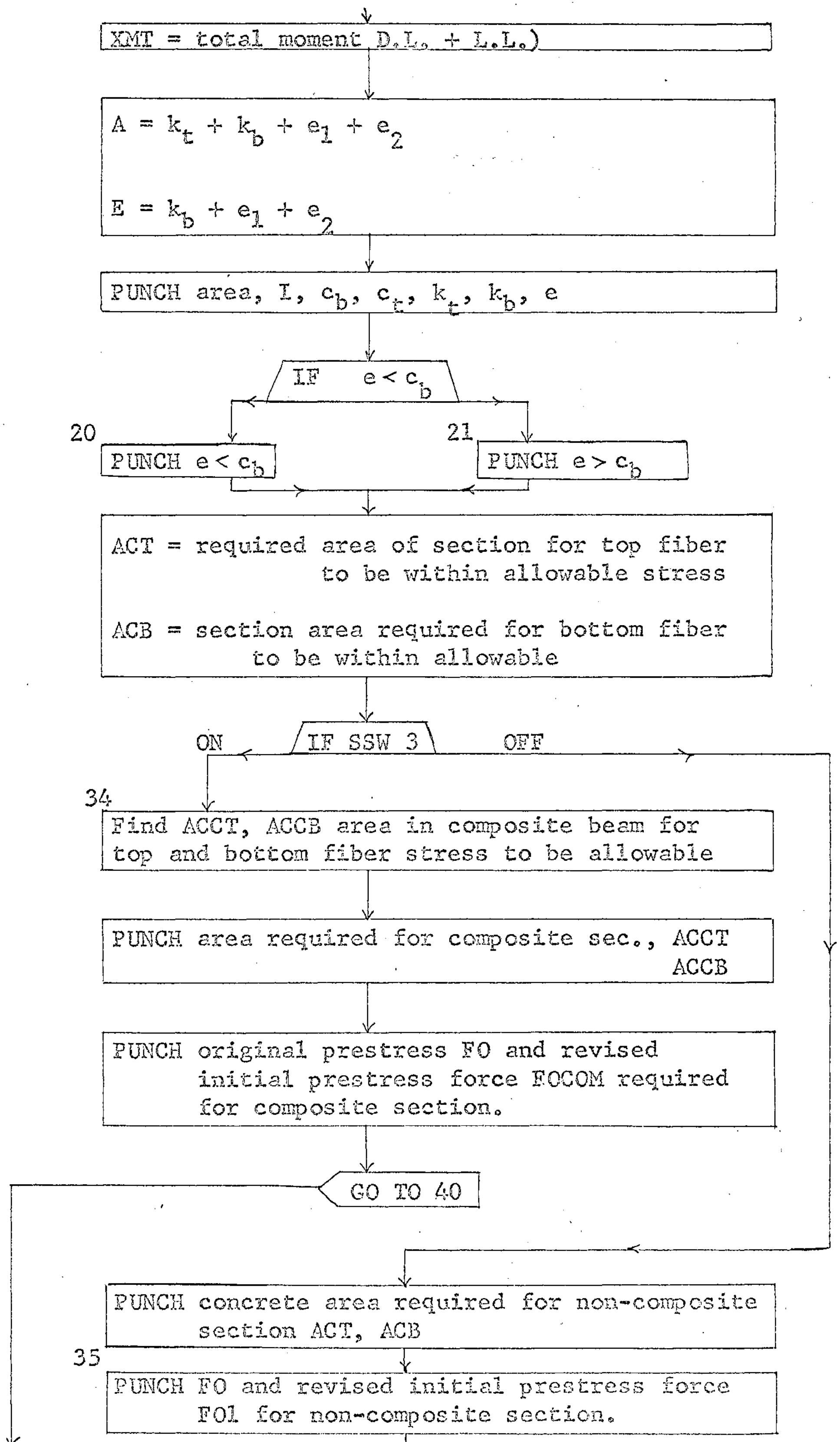


Figure 40. Continuation of Flow Chart.

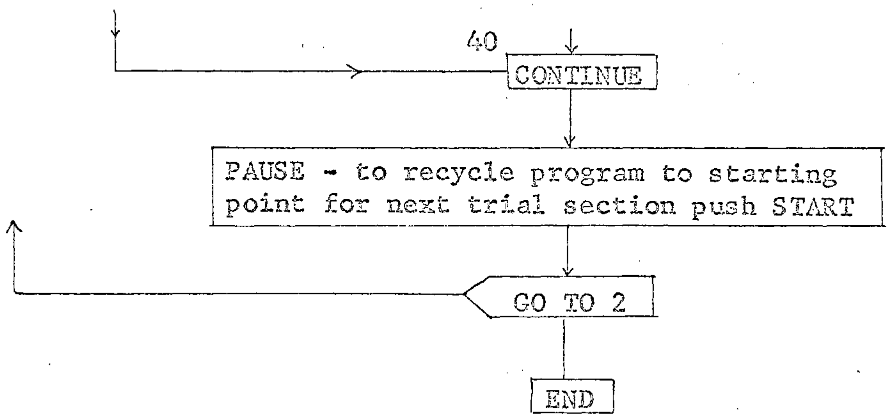


Figure 41.

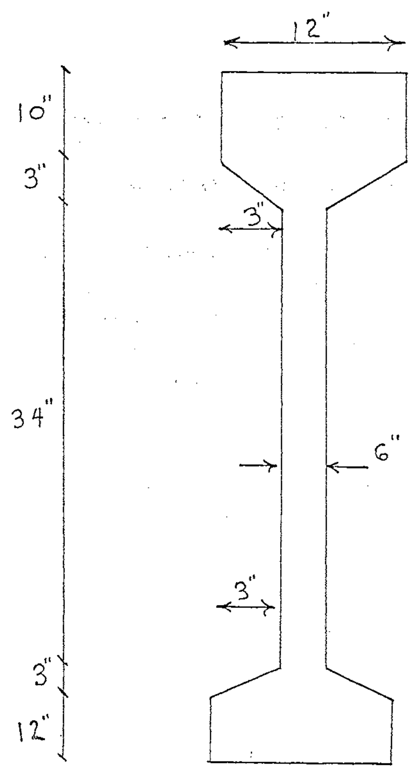
Continuation of Flow Chart

SAMPLE PROBLEM INPUT DATA

60.0	12.0	10.0	12.0	12.0	6.0	3.0	3.
3.0	3.0	4930.00	13870.00	555.0	.20	4.0	6.

OUTPUT DATA

PRECAST SECTION PROPERTIES - TENSION ALLOWED
OUTPUT DIMENSIONS ARE IN INCHES, IN**2, IN**4, AND KIPS
AREA = 510.000 I= .19410847E+06 CB= .29588235E+02
CT = .03411765E+02
KT= 12.863 KB= 12.515 E = 23.579
E IS SMALLER THAN CB - O.K.
FO = 555.000 FO1 = 540.257
AC(BOT) = 481.125 AC(TOP) = 394.383



F_o = 555.0^k

M_G = 4930 in.-k

$M_{L.L.}$ = 13,870 in.-k

Prestress Loss = 20%

f'_{ci} = 4.0 ksi.

f'_c = 6.0 ksi.

Figure 42.

NONCOMPOSITE "I" SECTION WITH TENSION ALLOWED

The output data reveals that the area of the section is 510.0 sq. inches. The moment of inertia is 194,108.47 inches. The c.g.c. 29.58 inches up from the bottom fiber and 30.41 inches down from the fiber. The top kern point is 12,863 inches up from the c.g.c. and 12.515 inches down from the c.g.c. is the bottom kern point. The ecc tricity is 23.579 inches which is smaller than the distance from c.g. to the bottom fiber so that the cable is in the beam. Initial trial stress force was 555.0 kips and the revised value is 540.257 kips. Tl required beam area for stress at the bottom fiber to be within the A. allowable was 481.125 sq. inches, and for allowable stress at the top fiber a 394.383 sq. inch section is required. Both required areas we less than that provided by the trial section. This trial section woul be adequate to sustain the applied bending moments. Other problems such as shear would have to be checked before final approval of the se tion, of course.

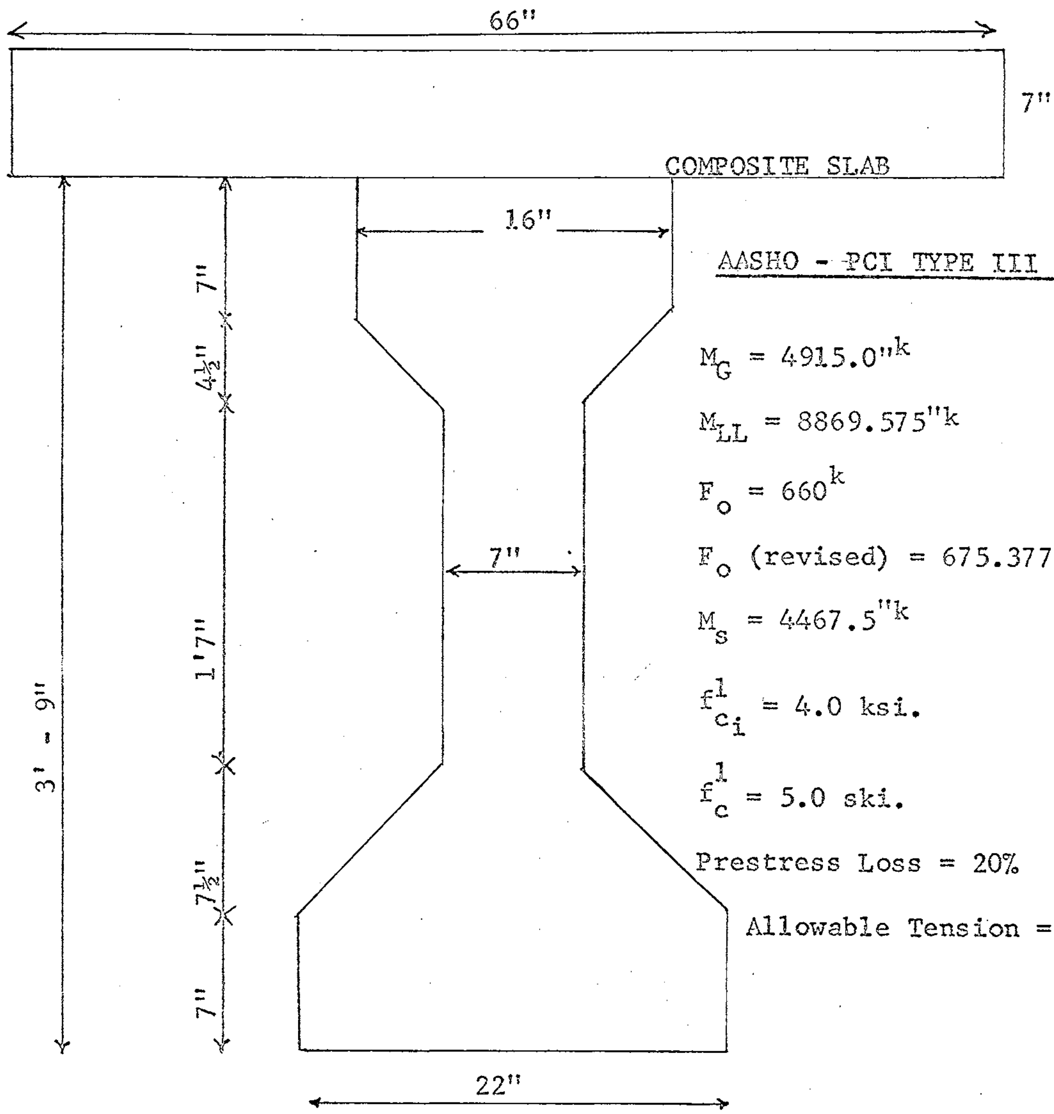
INPUT DATA

45.0	16.0	7.0	22.0	7.0	7.0	4.5	4.5
7.5	7.5	4915.0	8869.575	660.0	.20	4.0	5.0
4467.5	66.0	7.0					
0.0	0.0						

OUTPUT DATA

```
PRECAST SECTION PROPERTIES  -  TENSION ALLOWED
OUTPUT DIMENSIONS ARE IN INCHES, IN**2, IN**4, AND KIPS
AREA=   559.500     I =  .12539030E+06    CB- .20273458E+02
                    CT= .24726542E+02
KT=   11.054    KB=       9.063   E =     16.510
E IS SMALLER THAN CB  -  O.K.
COMPOSITE SECTION REQUIREMENTS
FO=    660.000    FOI=     675.377
AC(BOT)=    516.451   AC(TOP)=   393.228
```

The output shows that the required area is 516.451 sq. inches, which is less than the 559,500 sq. inches provided. Thus, the section is satisfactory.



AASHO - PCI TYPE III BEAM

$M_G = 4915.0''^k$

$M_{LL} = 8869.575''^k$

$F_o = 660^k$

F_o (revised) $= 675.377^k$

$M_s = 4467.5''^k$

$f^1_{c_i} = 4.0$ ksi.

$f^1_c = 5.0$ ski.

Prestress Loss = 20%

Allowable Tension = 0.0 k

Figure 43.

AASHO-PCI TYPE III BEAM - NO TENSION ALLOWED[4]

[4] H. Kent Preston, PRACTICAL PRESTRESSED CONCRETE, (McGraw-Hill, Publisher), Chapter 9.

Solution of Systems of Simultaneous Equations

If the simultaneous equations for a slope-deflection solution of a problem are set up manually, this will necessitate the solution of the system of equations. When simultaneous equations grow beyond four unknowns, the arithmetic becomes cumbersome, and a computer solution is advantageous. There are numerous ways to solve systems of simultan ous equations, and many programs are available for this purpose.

One of the most common methods of solution is the Gauss-Jordan elimination method. The method basically involves eliminating all of the elements in the coefficient matrix of the equations, except for th main diagonal elements whose coefficient value is reduced to one. Thi is done by performing various arithmatic operations on the rows of the matrix. A typical set of simultaneous equations is shown below, with their corresponding augmented coefficient matrix.

$$X_1 + X_2 + X_3 = 9$$
$$2X_1 + 3X_2 - X_3 = 6$$
$$2X_1 + X_2 - X_3 = 0$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & 3 & -1 & 6 \\ 2 & 1 & -1 & 0 \end{array}\right]$$

The aumented matrix is operated on to make the left partition of the matrix into a unit matrix, with all elements equal to zero except the main diagonal which becomes all one's. The final values of the right side of the partitioned matrix become the values of the unknown

A complete explanation of the manipulations involved in solving set of simultaneous equations by the Gauss-Jordan method may be found most books on matrix algebra. The augmented matrix shown on the prev page is solved manually below. The symbol "R" stands for row (of the matrix).

$$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 2 & 3 & -1 & 6 \\ 2 & 1 & -1 & 0 \end{bmatrix} \begin{array}{c} \longrightarrow \\ R_2-2R_1 \\ R_3-2R_1 \end{array} \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & -3 & -12 \\ 0 & -1 & -3 & -18 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & -3 & -12 \\ 0 & -1 & -1 & -18 \end{bmatrix} \begin{array}{c} \overrightarrow{R_1-R_2} \\ \\ R_3+R_2 \end{array} \begin{bmatrix} 1 & 0 & 4 & 21 \\ 0 & 1 & -3 & -12 \\ 0 & 0 & -6 & -30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 4 & 21 \\ 0 & 1 & -3 & -12 \\ 0 & 0 & -6 & -30 \end{bmatrix} \begin{array}{c} \overrightarrow{R_1+2R_3/3} \\ R_2-R_3/2 \\ -R_3/6 \end{array} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$X_1 = 1$

$X_2 = 3$

$X_3 = 5$

The Gauss-Jordan elmination method has been programmed, and some slight modifications have been added to fascilitate accuracy.[1] A general matrix array A(K,J) is read into the computer for the elements of the augmented coefficient matrix. For N equations, the size of the array is A(N,N+1).

In performing the elimination process, the diagonal element in the row "k", A(k,k), plays a particularly important role, and it is often called the pivot element. Since one of the steps is to divide row "k" by the element A(k,k), it is mandatory that the pivot element not be zero. If a zero pivot element occurs at any step, an attempt may be made to exchange the row containing the zero with a row below it in which the element in that column is not zero. Likewise, if a pivot element is not zero, but is small, it may contain a large relative error. This situation may be remedied, to a certain extent, by searching at each step for the element of greatest magnitude in the pivot column (in the pivot row and those below) and exchanging rows so as to use it as the pivot element. After the array A(N,N+1) has been read in, a routine is used to exchange the rows (or equations) of the matrix so as to maximize the pivot elements A9k,k) and to make sure that none of the pivot elements are equal to zero. After this shuffling of rows has been completed, then the Gauss-Jordan elimination is begun. A flow chart for the elimination procedure is presented on the next page.

[1] Raymond W. Southworth and Samuel L. Deleeuw, Digital Computation and Numerical Methods, (McGraw-Hill Book Company, 1965).

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[1] Raymond W. Southworth and Samuel L. Deleeuw, <u>Digital Computation and Numerical Methods</u>, (McGraw-Hill Book Company, 1965).

TABLE XX

OUTPUT DATA FOR SEVEN SIMULTANEOUS EQUATIONS

ACTUAL EQUATIONS

$$
\begin{aligned}
X_1 + 3X_2 + X_3 \qquad + X_5 + 2X_6 + 4X_7 &= 1 \\
+ 2X_2 \qquad + X_4 \qquad + 2X_6 \qquad &= 3 \\
3X_3 + X_4 + X_5 \qquad + X_7 &= 5 \\
2X_2 \qquad + 2X_4 \qquad + 4X_6 + 3X_7 &= 1 \\
2X_1 + 6X_2 + 2X_3 \qquad + 6X_5 + 5X_6 + 9X_7 &= 3 \\
X_1 + 5X_2 + X_3 + X_4 + X_5 + 5X_6 + 6X_7 &= 7 \\
X_1 \qquad + 4X_3 + X_4 + 2X_5 + 2X_6 + 8X_7 &= 8
\end{aligned}
$$

MODIFIED GAUSS-JORDAN ELIMINATION FOR SIMULTANEOUS EQS.

```
X 1 =      8.833
X 2 =     -7.000
X 3 =      9.250
X 4 =    -14.333
X 5 =     -2.083
X 6 =     15.666
X 7 =     -6.333
```

Figure 45.

Sample Problem for Solution of Simultaneous Equations

CHAPTER VII

DISCUSSION AND CONCLUSIONS

The practical application of computers in structural work is no longer only a matter for the researcher to investigate. It is a reali in the top design offices today. A recent survey of the actual amount of structural work being done by computer in commercial concerns shows this to be the case.[1] Each year, Engineering-News Record magazine doe a survey to determine who the top 500 consulting engineering firms in the nation are. Included in the questionnaires sent to over 7,000 consulting firms by Engineering-News Record were questions pertaining to the extent of computer use done by each consultant. The results of the questions concerning computer usage have been summarized.[1]

> "Three out of five of the straight design firms on ENR's list of the Top Design Firms use electronic computers.
>
> Among these(29) firms who have annual billings of \$5 million or more, computer users outnumber non-users by a 5:1 ratio. In the next lower billing bracket, \$2.5 million to \$5 million, (43 firms) ratio of computer users zooms to 13:1. But the ratio slips back as billings volume declines below that point. In the \$1 million to \$2.5 million billings bracket (146 firms), computer users out-number nonusers 3:2.
>
> Not until billings volume drops below \$1 million does the computer fail to win a distinct majority of the Top Design Firms. But, even at that, 48% of the (238) firms in that bracket use computers."

[1] Elwyn H. King, Electronic Computation: Past, Present, Potential, (Vol. 92, NO. ST6. Dec., 1966), Journal of the Structural Division, Proceedings of the American Society of Civil Engineers.

Below the bracket of the top 500 consulting firms, many of the smaller firms have special computer consultants available to them. Many small firms are renting small amounts of time on the computers of larger firms. "Time sharing" is becoming popular. Here, small users may hook into large computers thousands of miles away, on the telephone. An example of this is the many users in South America who regularly buy time over the telephone on the I.B.M. 7094 computer at M.I.T. in Cambridge, Massachusetts. Many firms have found that they do not have to own a computer to use one. In contract, the consulting firm of Gauger Engineering, Tulsa, Oklahoma, is a "one man firm" which has its own I.B.M. 1130 computer. Fred Gauger is able to turn out large structural designs in a minimum amount of time with his computer. A solid example of his work is a new 32-story building which he designed on his computer.

In short, computers are being used extensively in design firms today. The new I.B.M. 1130 computer can be leased for about $1,000.00 per month (or more, if you want extra features such as a high speed printer), and this "third generation" computer has begun to appear in design offices everywhere. The 1130 is faster, larger, and cheaper to operate than the "second generation" 1620, which is now obsolete. Of the major consulting firms which the author of this report has talked with, most have an 1130, or have one on order. The Engineering-News Record survey disclosed that 49 out of the 50 state highway departments in the United States use an electronic computer in their work. There are some stubborn opponents left, but their number is dwindling, perhaps like their profits.

How many and what types of programs are being used by practicing engineers? This question was asked by the Engineering-News Record survey. It was found that the average program library of a firm contained about 28 programs.

> "In response to the request to list the five most used and/or valuable structural programs, 80 replied and made 286 citations of more than 90 programs that have identical or similar names or were obviously similarly-purposed programs. Abstracts were requested, but too few were submitted to permit analysis."[2]

It was found that the great majority of programs were of the "workhorse" type (like the 'special purpose' programs presented in Chapter VI of this report), and few were as complex as the space frame program presented earlier. Mostly routine calculations were programmed, and only a few programs could be considered complex or capable of performing any task which an engineer would hesitate to do manually. The practicing engineer takes a very practical approach to programming, and programs those things which will directly save him time and make him money. The more exotic problems are left to the researchers.

A table is given on the next page which gives a breakdown of the 286 programs which engineers said that they used most commonly.[2]

[2] E. H. King, Electronic Computation: Past, Present, Potential, Journal of the Structural Division, (Proc. of the A.S.C.E., Dec., 1966), p. 11, Table 4.

TABLE XXI

MOST USED AND/OR MOST VALUABLE PROGRAMS

Bridge Pier Analysis (including Rigid Frame)	(20)[3]
Frame Analysis (Steel, Concrete)	(23)
Multistory Rigid Frame Analysis	(3)
Lateral (including E Q) and Vert. Loading on High Rise Buildings	(6)
Space Frame Analysis	(2)
Continuous Truss Analysis	(3)
Turss (Analysis and Design (Determinate or Indeterminate)	(7)
STRESS	(20)
Transmission Tower Analysis	(3)
Curved Beam Analysis	(2)
Composite Beam Design	(18)
Composite Welded Girder Design	(13)
Concrete Beam Analysis	(3)
Prestressed Concrete Beam Design	(6)
Analysis of Beams Subject to Torsion Beam and Girder Designs	(2)
Floor Framing Design and Analysis	(6)
Flat Slab Design	(2)
Analysis of Slabs and Shells	
Steel Detailing	(2)
Spiral Stairs	
Chimney Design or Analysis	
Closed Ring Analysis	
Arch Dam Analyses, Deflection and Stresses	(2)
Elastic Arch Analysis	(2)
Analysis of Sewer Arches and Rings by Voussoirs	(2)
Foundation Designs-Rectangular Trapezoidal, Combined Ftgs; Mats	(4)
Sheet Piling Design	
Beams on Elastic Foundations	
Retaining Walls	
Laterally Loaded Pile Groups	
Substructure Analysis	
Structural Geometry	
Bridge Geometry (horiz.,) vertical, skewed)	(
Bridge Deck and Bearing Elev.	
Curved Bridge Geometry	
Framing Plan	
Influence Lines	
Continuous Beam Moment Envelope (AASHO)	
Continuous Beam Analysis, Design, Detail	(
Design or Analysis of Circular R/C Columns	
Rectangular R/C Col.Des.	
Ultimate or Elastic Column Analysis and Design	
Steel Column-Loads and Design	
Beam Moments, Shears and Deflection	
Properties of Sections	
Piping Flexibility and Stresses	
Various Services Bureau Proprietary Routines	

[3] Numbers in parentheses indicate number of citations when greater tha one.

Up to the present, most programming has been in the field of anal; sis, with less emphasis on the concept of design of structures. The Third Conference on Electronic Computation was held by the Committee on Electronic Computation of the Structural Division of ASCE in Boulder, Colorado, June 19-21, 1963. As in the first two conferences (dating back to 1958), theory and analysis were the main topics of dis- cussion. In the third conference, methods of analyzing very complex structures were discussed. Most programs available to date are analysi programs, and most of the programs in this report dealth with analysis.

The Fourth Conference on Electronic Computation (University of California, at Los Angeles, September 7-9, 1966) emphasized the design aspects of electronic computation. Design is the area for new develop- ment in programming. The only example of true design in this report i the cover plate design feature in LINK 9 of the highway bridge program. Here, increments are added to the cover plate area until the stresses are within the allowable for the moments present. The prestressed con- crete program has been used successfully by the author in rapidly check- ing and redesigning prestressed concrete sections. As mentioned prev- iously, seven trial and error runs were made in one-half hour to revis the design of a prestressed concrete cross-section. Here, the actual redesigning of the section was done by the author and the computer checked it.

An excellent commercial design program is FRAME. On special cod- ing sheets, an engineer describes the geometry of a building (where the members are placed) and records the loading conditions present on the structure. This data may be sent by air mail to New York where the data is read into a 7094 computer. A trial structure is set up and analyz From the moments and shears determined from the trial analysis, some new member sizes are chosen (All members in the A.I.S.C. Manual are stored in the computer memory). By trial and error, the computer determines the final sections, and the results are sent back by retur mail, for a fee. Service bureaus such as FRAME have been successful taking some business which an architect would have given to a structu engineer. As more consulting firms go into computer work, the servic bureaus will probably fade away.

Two other growing areas of development are automated preparation specifications and graphic display. Specifications writing is a high repetitive operation, and many sets of specifications are quite simil A large quantity of standard specifications may be read into the comp memory, and these to be used may be called out of memory and typed ou by the high speed printer. Consulting engineers are currently using these computer written specifications in the final contract. Graphic display is done by taking output of a computer program and feeding it into a plotting machine which plots the answers. The author recentl watched the loads applied to a tower foundation be read into an I.B.M 360 computer. From the applied loads, a foundation was designed by the computer. The answers were written on magnetic tape, and the tap was read into a plotting machine. In about four minutes, the entire plans for the foundation were drawn up on 24 x 36 drafting sheets, ir cluding the titles. Similarly, the framing plans and elevations for multi-story building were drawn on standard size drafting sheets in about five minutes. The computer selected the member sizes from the

A.I.S.C. Manual, and they appeared on the final drawings. The quality of the drawings was excellent. To prove the accuracy of the drawings, the magnetic tape was recycled through the automatic plotter, and the same picture was redrawn over the original drawing with no detectable difference in the drawing. It followed the original lines precisely. Auto-plotters are used by many highway departments to draw highway cros sections. The geometric coordinates of the highway cross-sections (which may be designed by computer) are read into the auto-plotter at the end of a work day, and the plotter does cross-sections through the night, shutting itself off when it runs out of data. The auto-plotter never will take the place of a draftsman on special projects, but it ma be applied to repetitive jobs.

The computer will not work miracles, but it will do much of the "hack work" for an engineer. It will permit the engineer to do a more thorough investigation of problems which were previously left to engine ing judgment, for lack of precise information. An engineer can get a better conception of what is happening in a multi-story building under wind load when he checks it with a space frame program than by manually analyzing with the cantilever method. The computer permits rapid inves tigation in depth, in the interest of safety and economy. An engineer may use his time much more efficiently with the aid of a computer.

> "Engineering practice by nature is a compromise between creativity and practicality, because the professional engineer has just so much time and money available to do his job.
>
> Using the computer as a tool, the engineer can more fully realize his design and professional output. He can examine more design possibilities

> before producing a final design and thereby strengthen the integrity of his finished work. The computer takes only a fraction of his time, and yet enables him to consider the many avenues available and refine his approach to design."[4]

The electronic computer can only do what it is told. It operates in a predetermined logical sequence, but it cannot think creatively. The creative thinking is left to the engineer, and for this reason, th computer will never replace the engineer. The engineer will always be required to solve all of the details which are not in the program.

A great deal of progress has been made in the past ten years in tl field of structural programming. The computer has become a practical tool for the structural engineer. He must grasp the tool and use it t his advantage.

[4] "Computer Refines Sewer Designs", Engineering-News Record, March 23, 1967.

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